Project Scheduling with Alternative Technologies: Incorporating Varying Activity Duration Variability
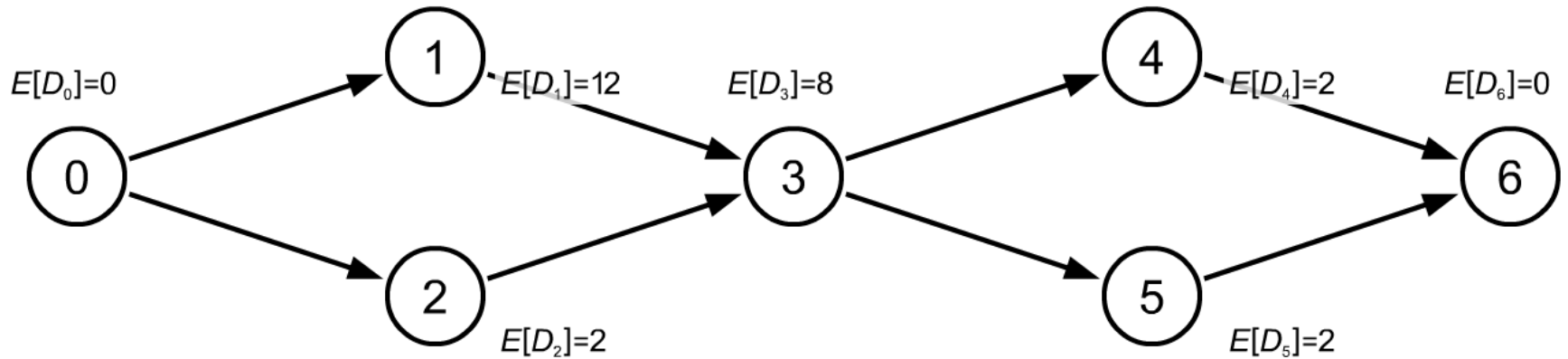
Stefan Creemers
Roel Leus
Bert De Reyck

- A project is a set of precedence-related activities that need to be completed in order to achieve a specific target

- Our objective is to schedule the activities of a project such that its value is maximized

- We examine how to incorporate the following characteristics:
  - Activity failure
  - Modular completion structure of the project
  - Different levels of variability in the durations of the activities

- Relevant especially for R&D and NPD but also in other sectors: pharmaceuticals, software development, …
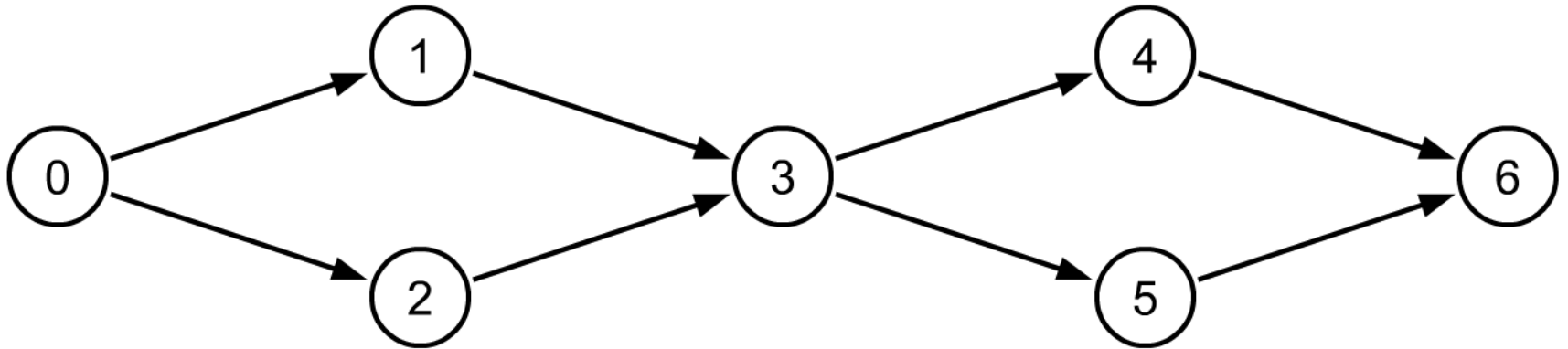
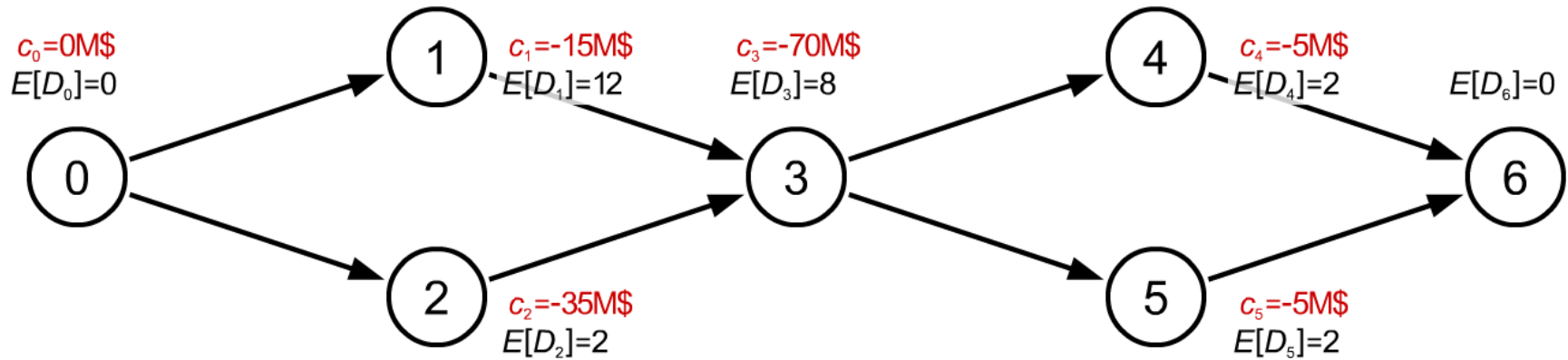- Project network with n activities (activity is on the node)

- Project network with n activities (activity is on the node)
- Stochastic activity durations: expected duration $E[D_j]$ of activity j

$c_0=0M\$$
$E[D_0]=0$

$c_1=-15M\$$
$E[D_1]=12$

$c_3=-70M\$$
$E[D_3]=8$

$c_4=-5M\$$
$E[D_4]=2$

$E[D_6]=0$

$c_2=-35M\$$
$E[D_2]=2$

$c_5=-5M\$$
$E[D_5]=2$

- Project network with n activities (activity is on the node)
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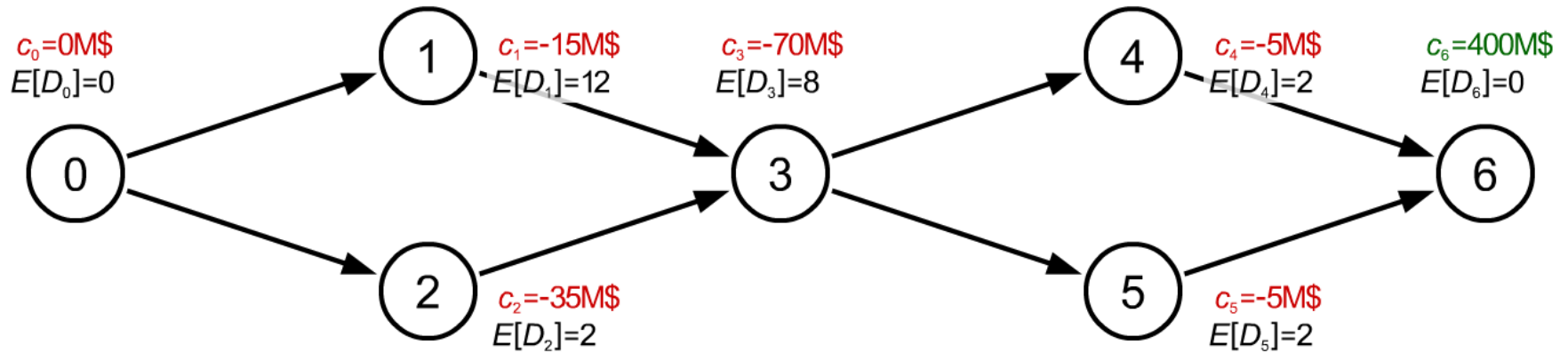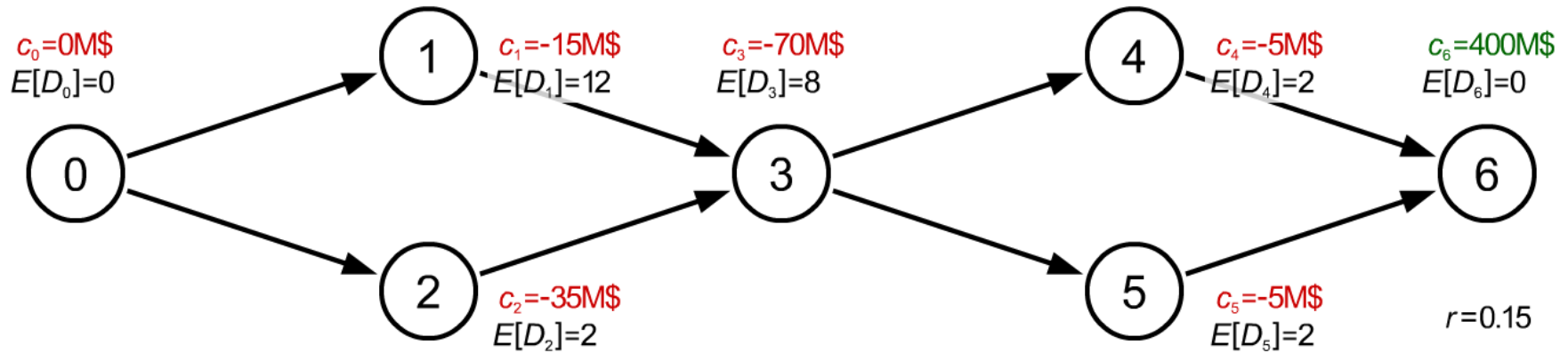- Cash flow c_j is incurred at the start of activity j

- Project network with *n* activities (activity is on the node)
- Stochastic activity durations: expected duration $E[D_j]$ of activity *j*
- Cash flow c_j is incurred at the start of activity *j*
- End-of project payoff *C* obtained upon overall project success

$c_0=0$M\$
$E[D_0]=0$

① 1
$c_1=-15$M\$
$E[D_1]=12$

$c_3=-70$M\$
$E[D_3]=8$

④ 4
$c_4=-5$M\$
$E[D_4]=2$

$c_6=400$M\$
$E[D_6]=0$

⓪ 0

③ 3

⑥ 6

② 2
$c_2=-35$M\$
$E[D_2]=2$
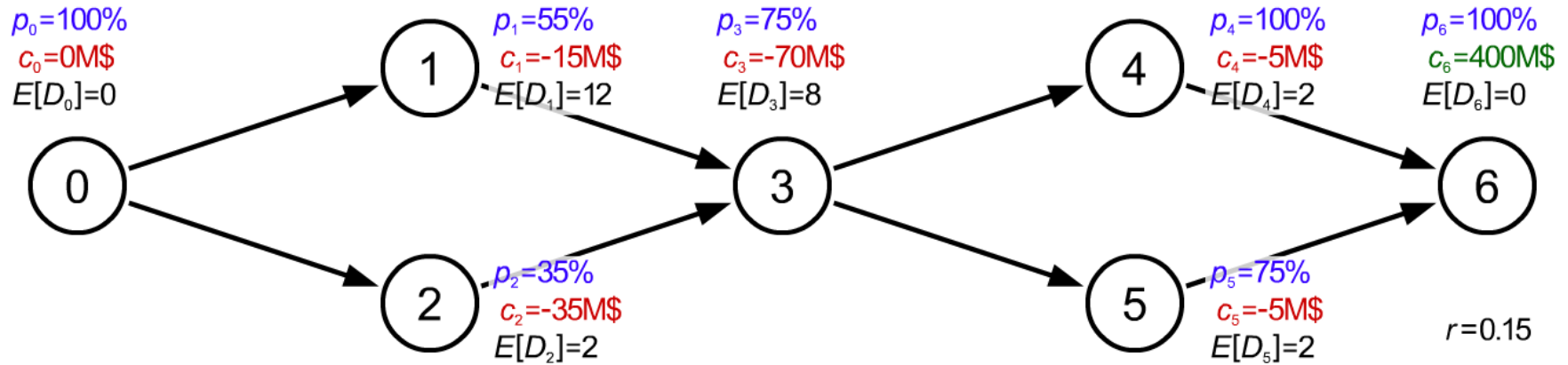
⑤ 5
$c_5=-5$M\$
$E[D_5]=2$

$r=0.15$

- Project network with n activities (activity is on the node)
- Stochastic activity durations: expected duration $E[D_j]$ of activity j
- Cash flow c_j is incurred at the start of activity j
- End-of project payoff C obtained upon overall project success
- Time value of money => discount rate r :
  the present value of a cash flow c after a time t is given by $c.e^{-rt}$

$p_0=100\%$
$c_0=0M\$$
$E[D_0]=0$

$p_1=55\%$
$c_1=-15M\$$
$E[D_1]=12$

$p_3=75\%$
$c_3=-70M\$$
$E[D_3]=8$

$p_4=100\%$
$c_4=-5M\$$
$E[D_4]=2$

$p_6=100\%$
$c_6=400M\$$
$E[D_6]=0$

$p_2=35\%$
$c_2=-35M\$$
$E[D_2]=2$

$p_5=75\%$
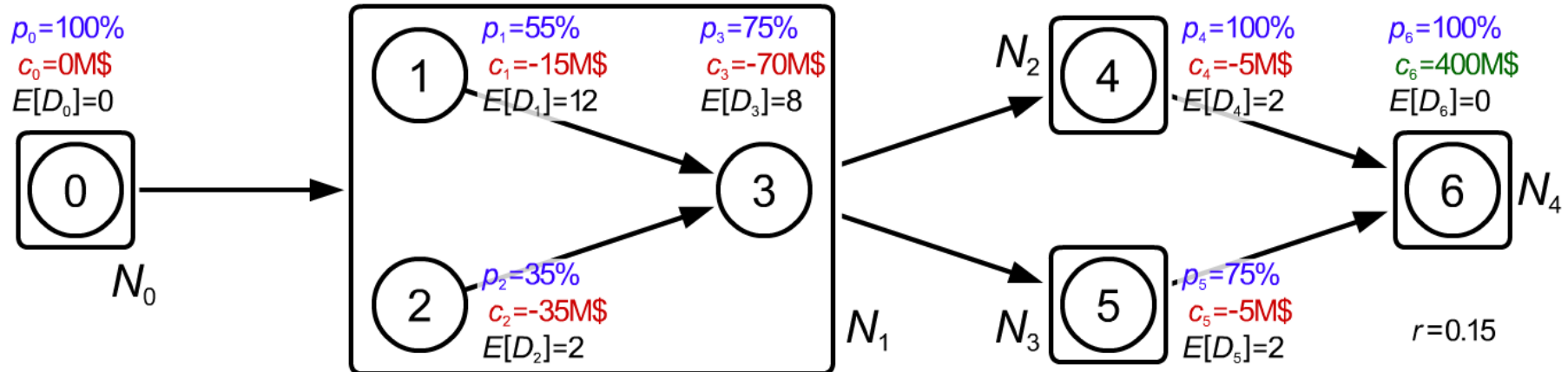$c_5=-5M\$$
$E[D_5]=2$

$r=0.15$

- Project network with n activities (activity is on the node)
- Stochastic activity durations: expected duration $E[D_j]$ of activity j
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- Failures: each activity j has a probability of technical success p_j

- Project network with *n* activities (activity is on the node)
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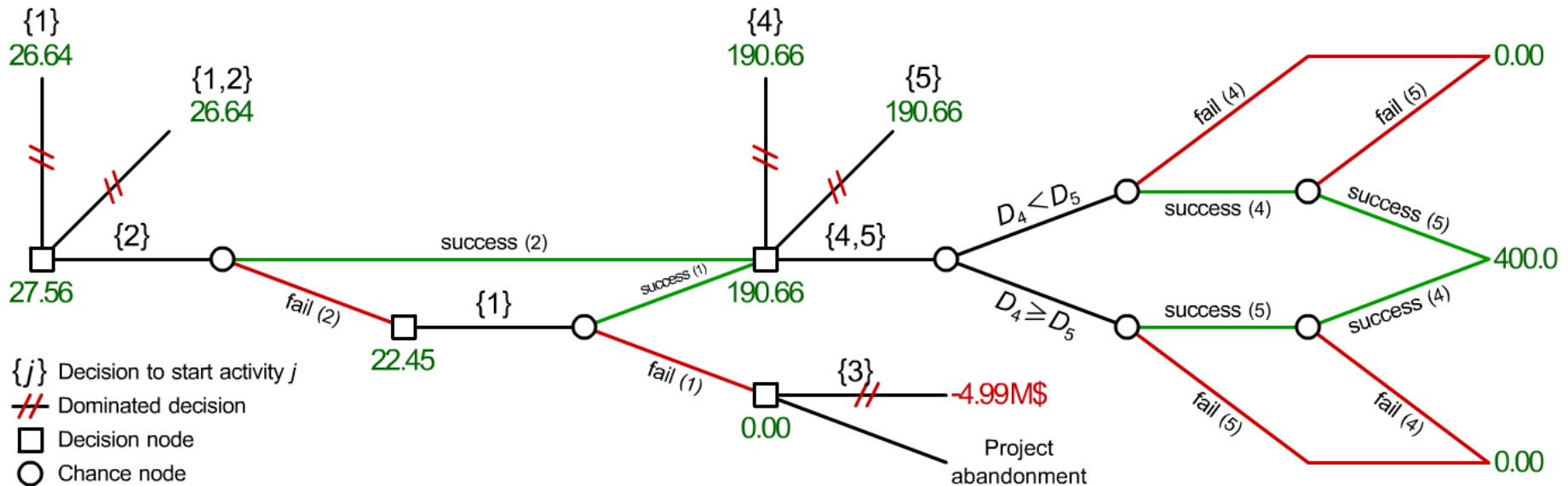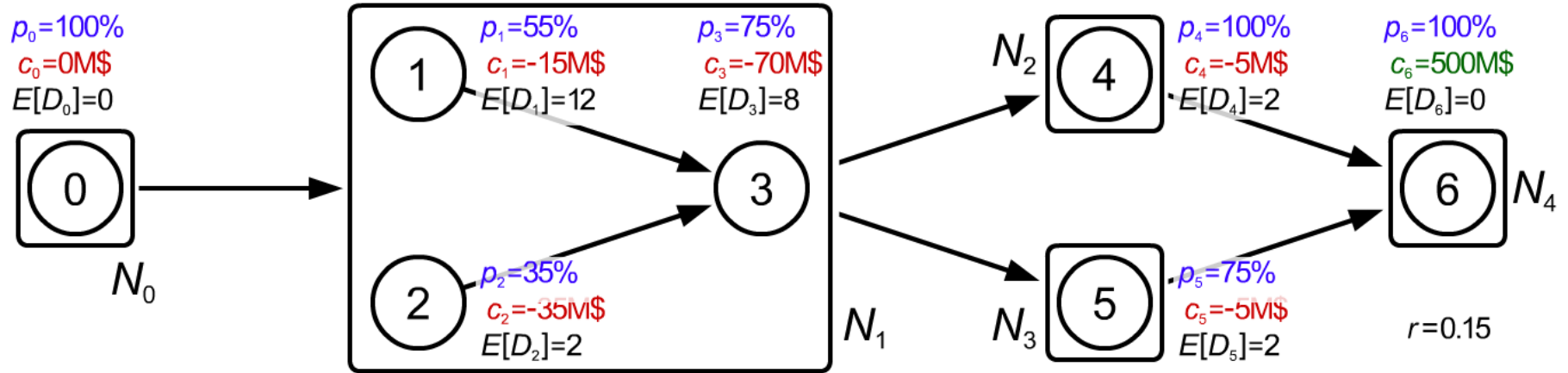- *m* modules N_i

SOLUTION METHODOLOGY

We build on earlier work of Creemers et al. (2010)*:

- At any moment in time, the state of each activity j can be:
    - Not Started
    - In progress
    - Past (successfully finished, failed or considered redundant because another activity of its module has completed successfully)

- The state of the system is defined by the state of the activities

- Use of Phase-Type distributions to model activity durations

- Use of a Continuous-Time Markov chain to model the statespace

- The optimal eNPV is found using a backward SDP-recursion

*Creemers S, Leus R & Lambrecht M (2010). Scheduling Markovian PERT networks to maximize the net present value. Operations Research Letters, vol. 38, no. 1, pp. 51 - 56.

p_0=100%
c_0=0M\$
$E[D_0]$=0

p_1=55%
c_1=-15M\$
$E[D_1]$=12

p_3=75%
c_3=-70M\$
$E[D_3]$=8

p_4=100%
c_4=-5M\$
$E[D_4]$=2

p_6=100%
c_6=500M\$
$E[D_6]$=0

p_2=35%
c_2=-35M\$
$E[D_2]$=2

p_5=75%
c_5=-5M\$
$E[D_5]$=2

N_0 N_1 N_2 N_3 N_4

r=0.15

{1}
26.64

{1,2}
26.64

{4}
190.66

{5}
190.66

{2}

success (2)

{4,5}

fail (4)

fail (5)

0.00

$D_4 < D_5$

success (4)

success (5)

27.56

success (1)

190.66

400.0

fail (2)

{1}

$D_4 \geq D_5$

success (5)

success (4)

22.45

fail (1)

{3}

-4.99M\$

fail (5)

fail (4)

0.00

Project
abandonment

0.00

{j} Decision to start activity j
⫻ Dominated decision
☐ Decision node
◯ Chance node

- What is the impact of the variability of activity durations on the eNPV of a project?

- Experimental setup:
    - All activities in the project have the same level of variability
    - Variability is expressed using the Squared Coefficient of Variation (SCV)
    - We observe SCV's ranging from 0 (deterministic) to ∞
    - We use Phase-Type distributions to model the activity durations*

Deterministic   Hypo-Exponential   Exponential   Hyper-Exponential

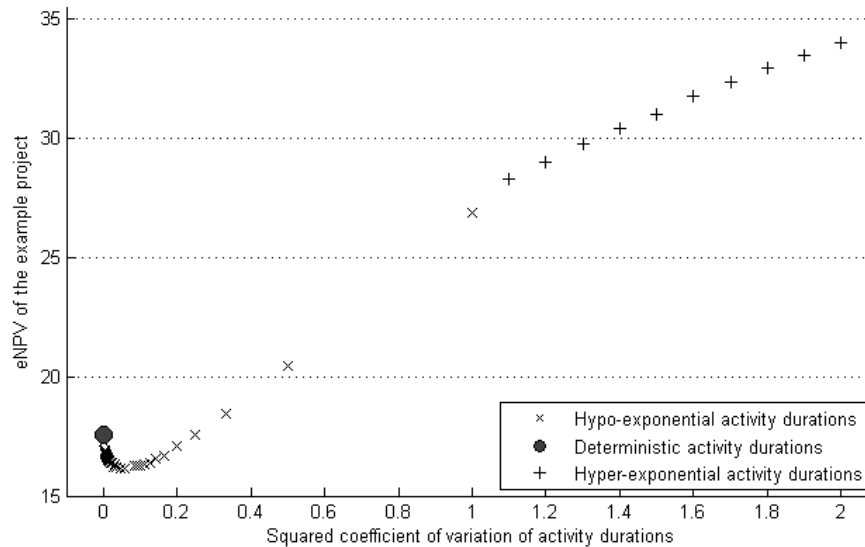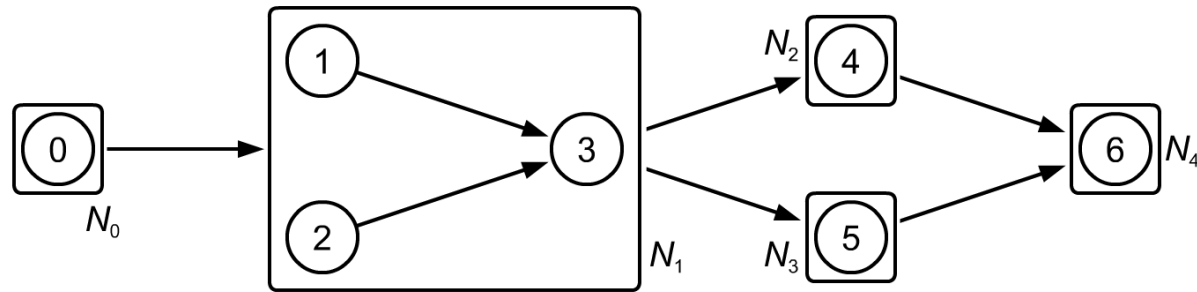SCV   0   1   ∞
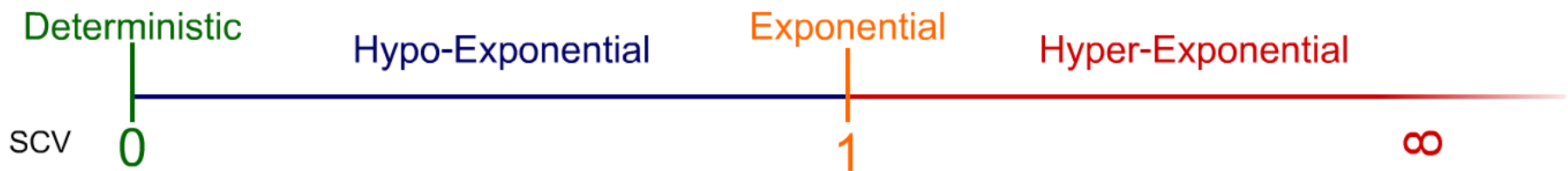
*We adopt the two-moment matching approximation of Creemers S & Lambrecht M (2010). An advanced queueing model to analyze appointment-driven service systems. Computers and Operations Research., vol. 36, no. 10, pp. 2773 - 2785.
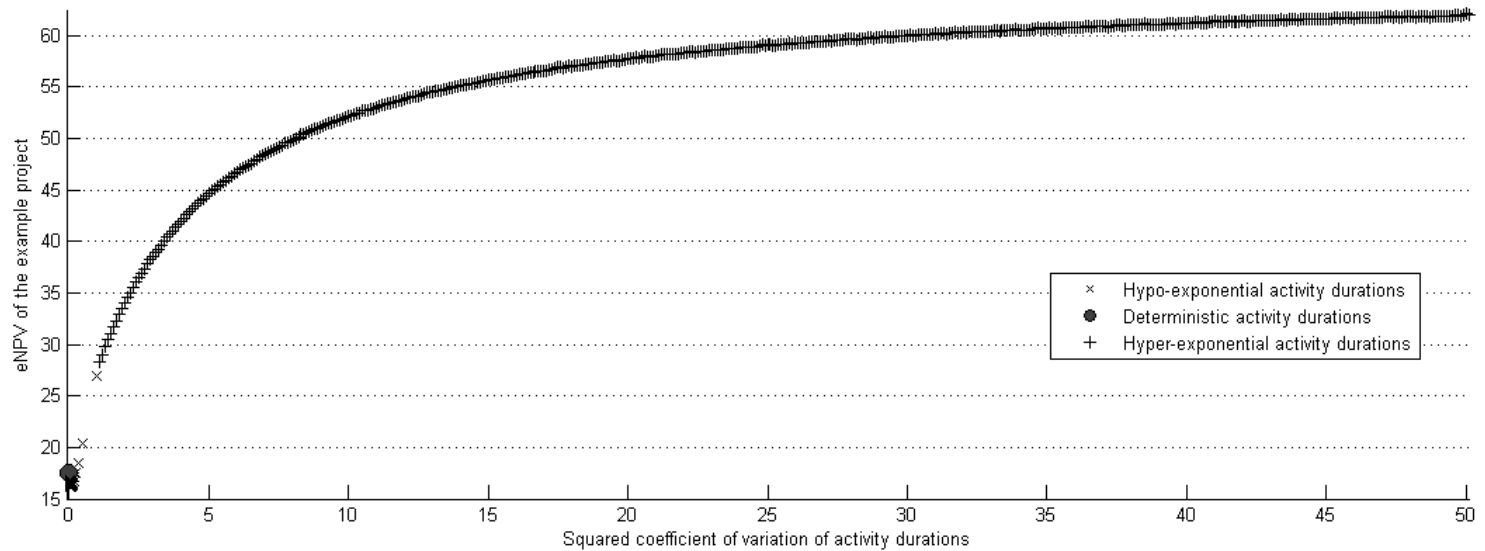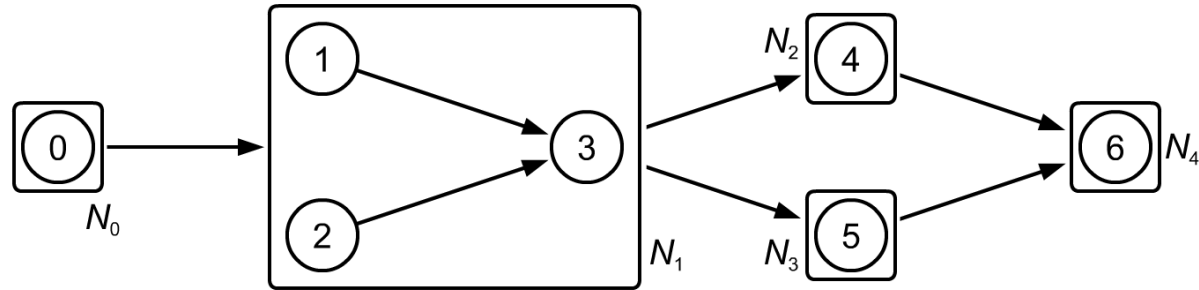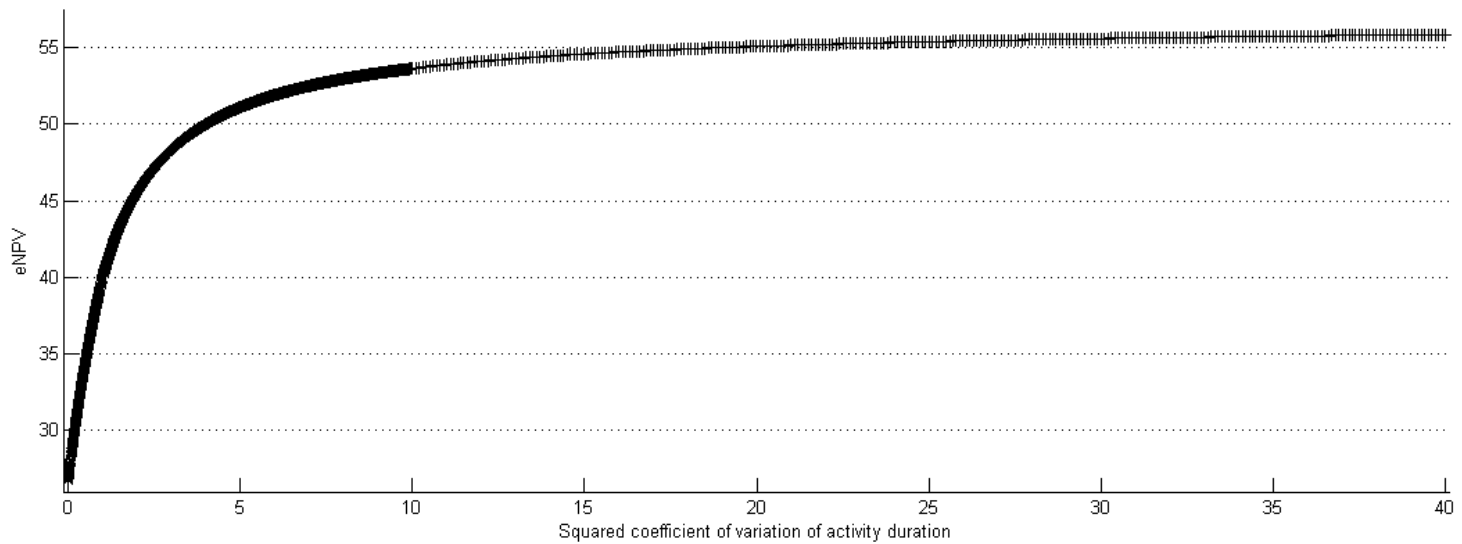
Computational results indicate that networks with up to 120 activities can be solved to optimality

SUMMARY

- We have extended the model of Creemers et al. (2010) to incorporate general activity durations (using Phase-Type distributions) & to allow for modular projects

- We have shown that variability in the duration of activities is not always bad with respect to the NPV of a project

- We have shown that even for a single activity, variability in the duration can be beneficial