Some recent advances in project scheduling

Stefan Creemers

(June 27, 2018)

IÉSEG SCHOOL OF MANAGEMENT

KATHOLIEKE UNIVERSITEIT LEUVEN

INTRODUCTION

Stefan who?

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Some example projects

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- Development of the Ebola vaccine

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- Development of the Ebola vaccine
- Organizing the FIFA World Cup

…

Project scheduling: important concepts

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What?

❓

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What?

Activities

Project scheduling: important concepts

What?

Who?

Activities

Project scheduling: important concepts

What?

Who?

Activities

Resources

Project scheduling: important concepts

What? **Who?** **When?**

Activities **Resources**

Project scheduling: important concepts

What?

Activities

Who?

Resources

When?

Schedule/Policy

Project scheduling: important concepts

What?

Activities

Who?

Resources

When?

Schedule/Policy

Why?

Project scheduling: important concepts

What?	**Who?**	**When?**	**Why?**
			
Activities	**Resources**	**Schedule/Policy**	**Makespan/NPV…**

Project scheduling problems we'll consider today

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- Minimize makespan:

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- Maximize NPV
  - Stochastic activity durations: **SNPV**
- All these problems are NP-hard!

RCPSP

(Resource-Constrained Project Scheduling Problem)

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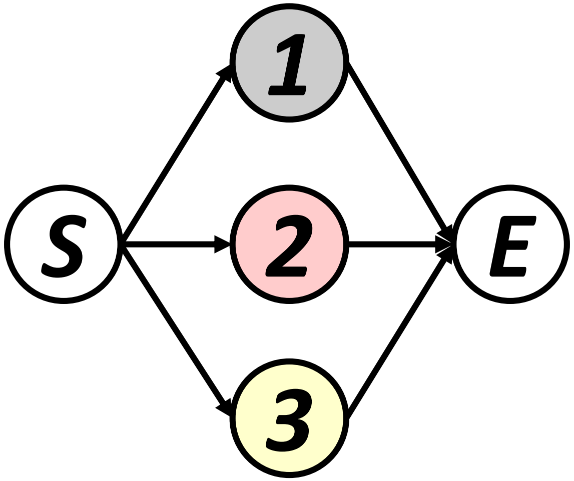

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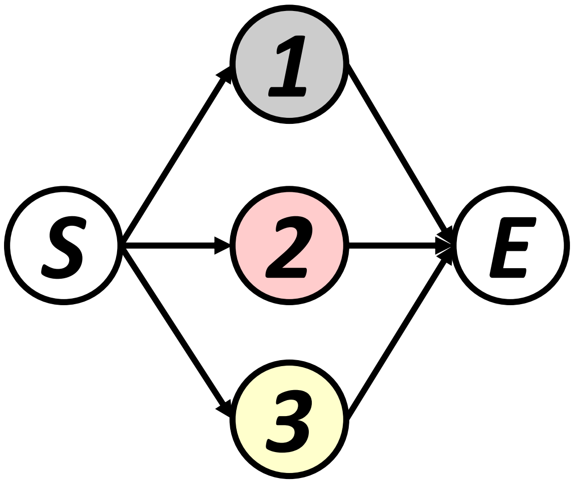

ACT	DUR	RESOURCE USE
1	2	1
2	2	1
3	2	1

Resource availability: 2

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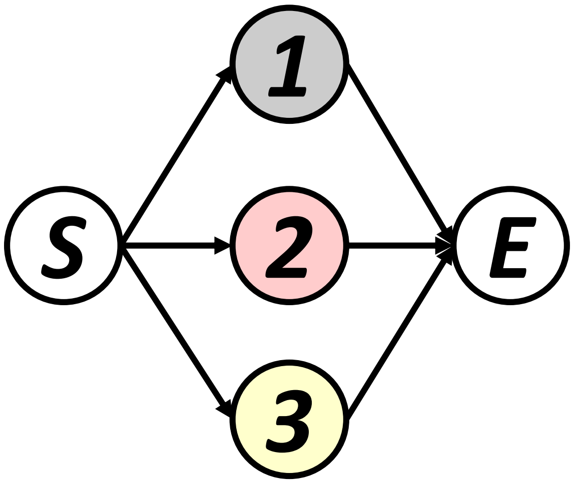

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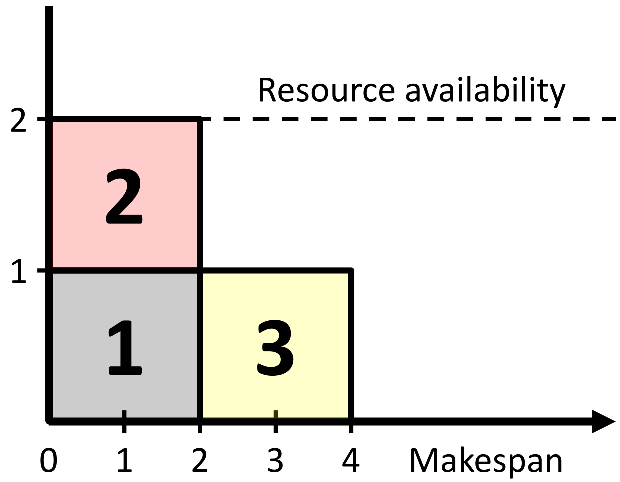
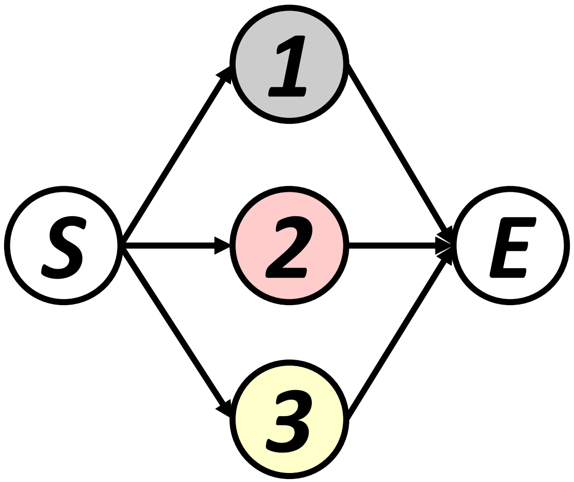


PRCPSP

(Preemptive Resource-Constrained Project Scheduling Problem)



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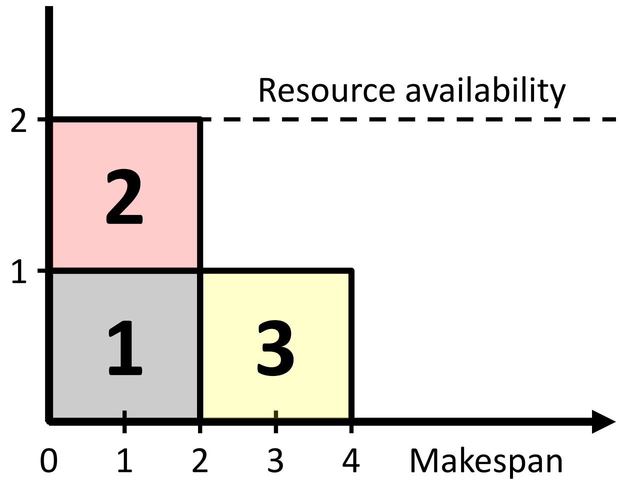

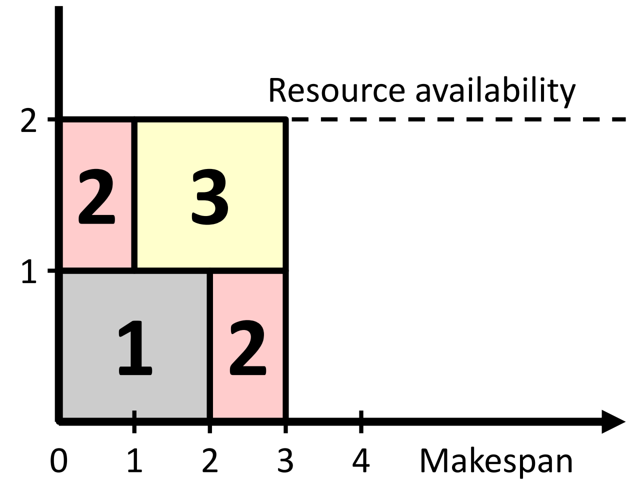

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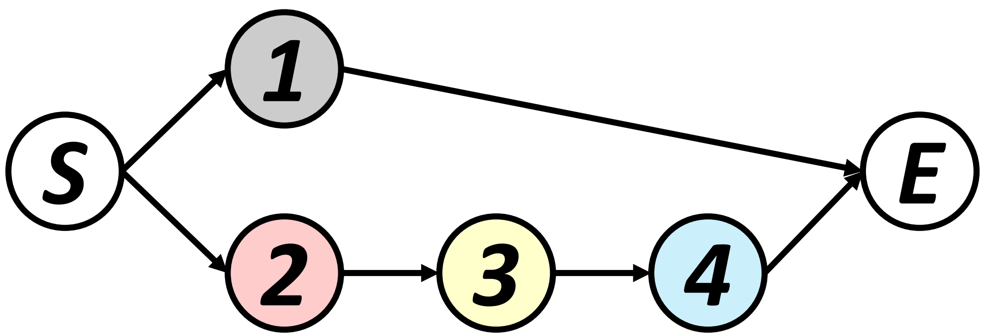

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3	2	2
4	2	1

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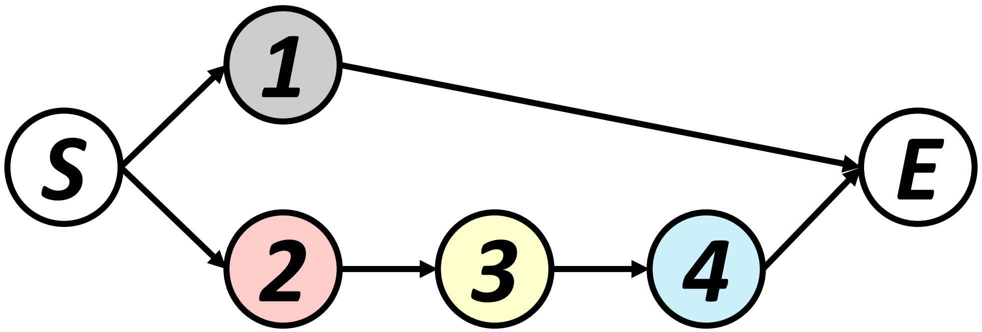

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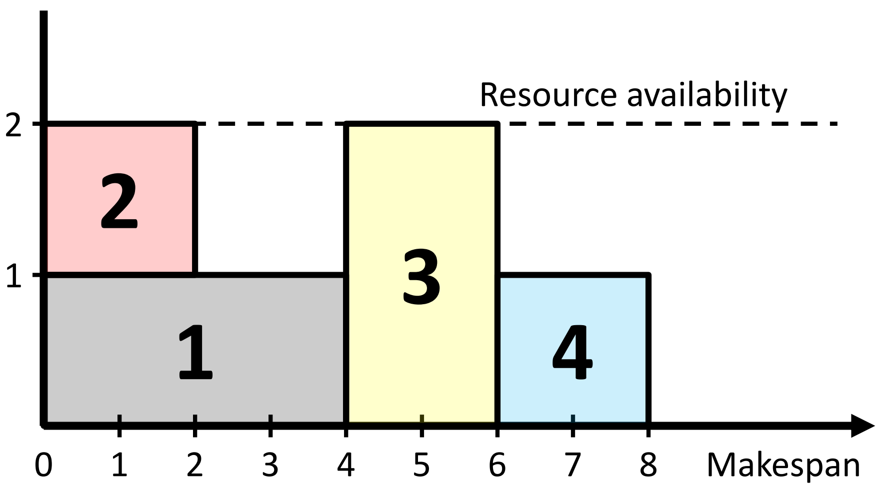

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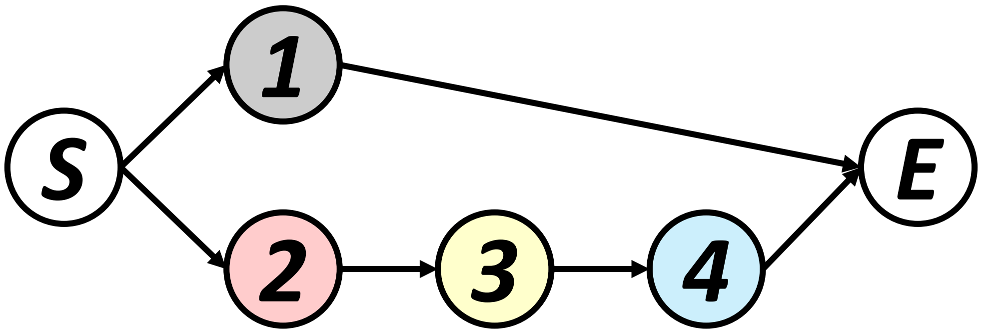

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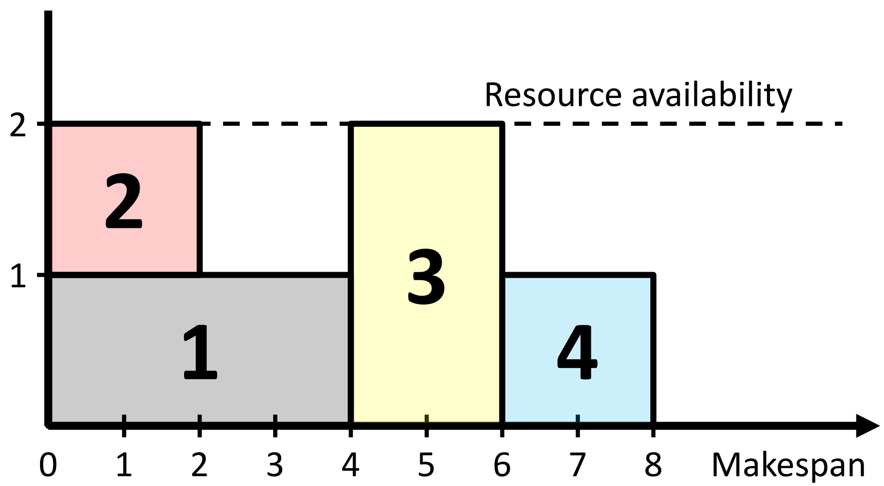
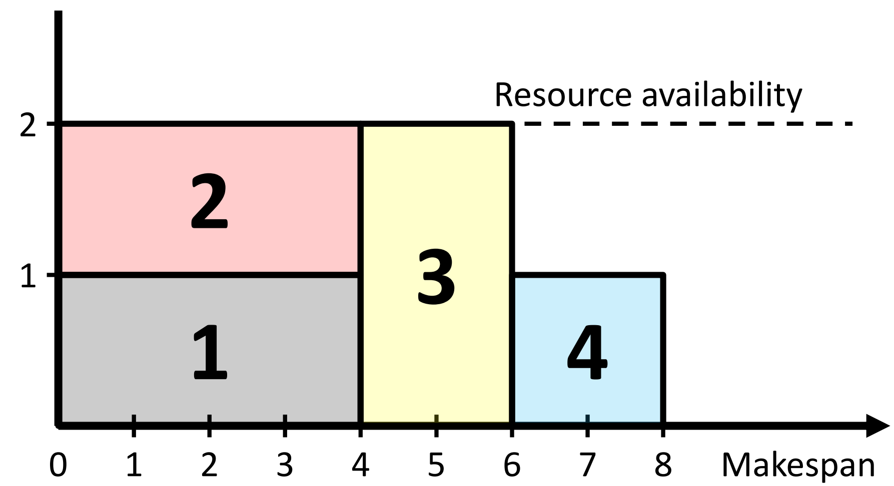
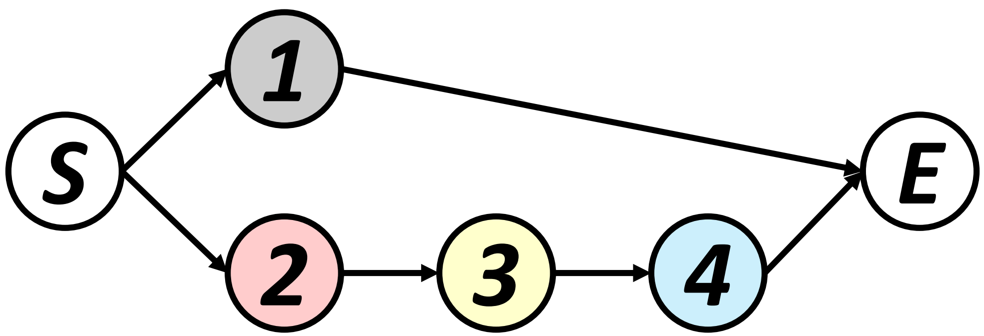




PSRCPSP

(Preemptive Stochastic Resource-Constrained Project Scheduling Problem)



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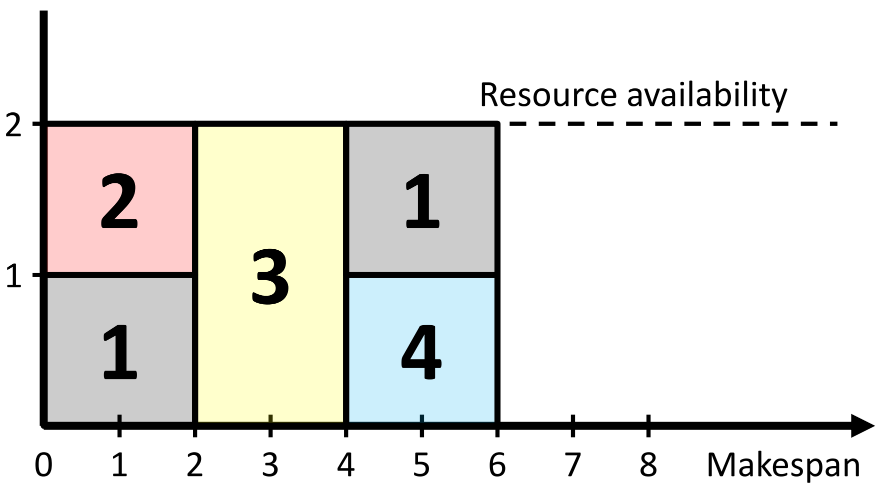

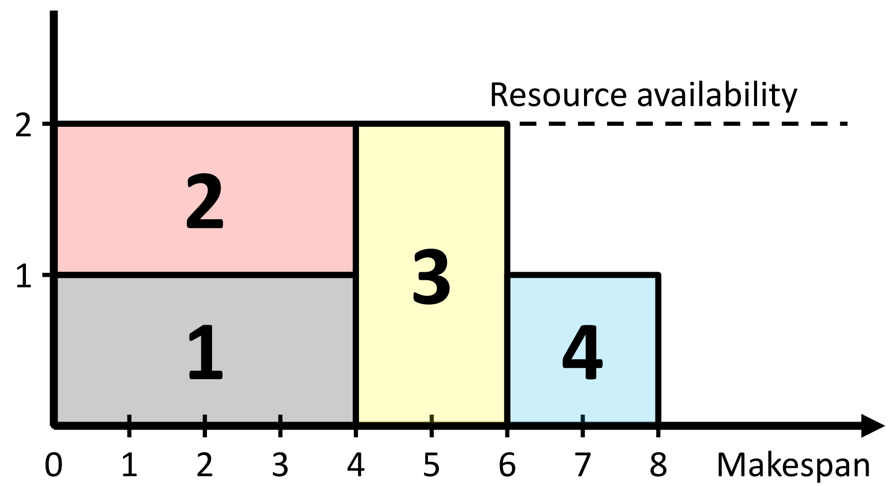

SNPV

(Stochastic expected NPV maximization problem)

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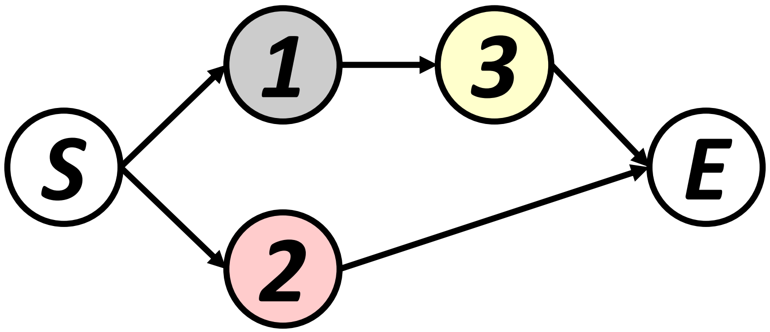

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Discount rate: 10%

Project payoff: 10

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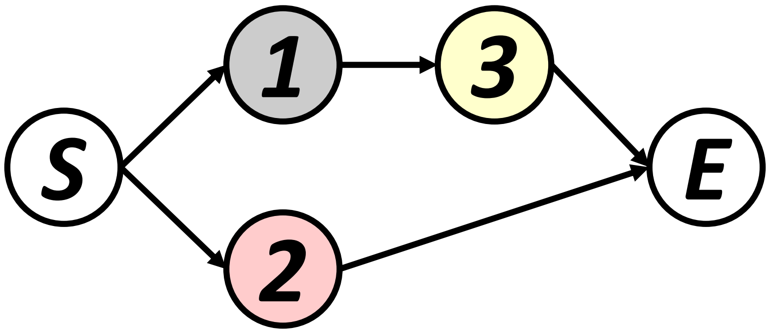

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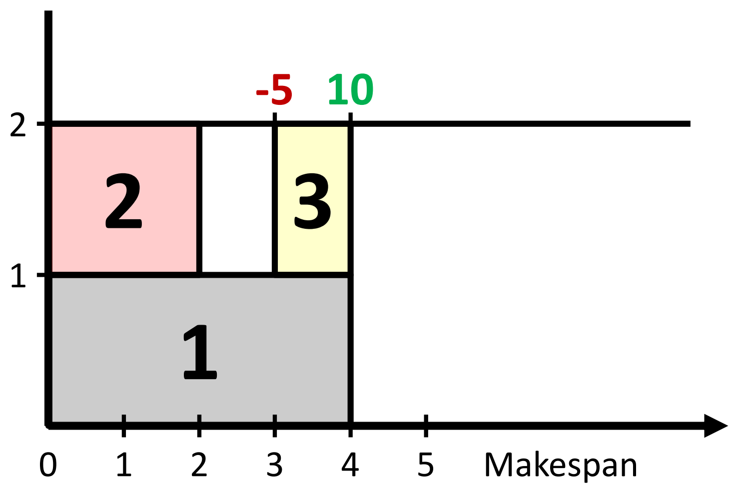

NPV = 3.00

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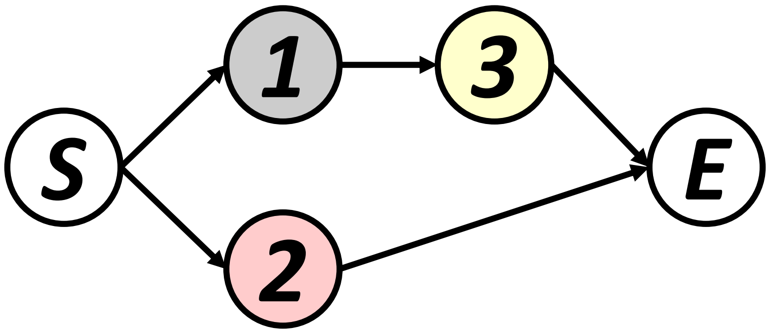

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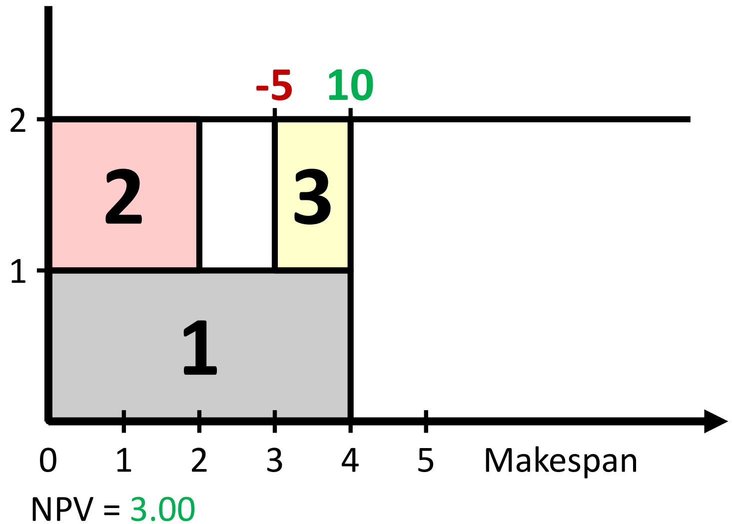

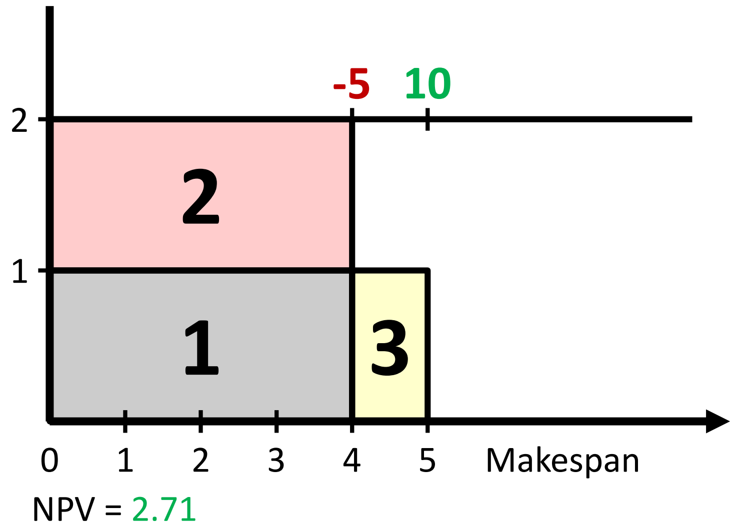

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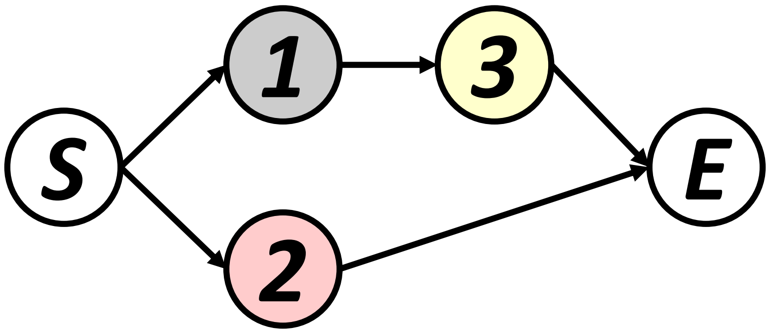

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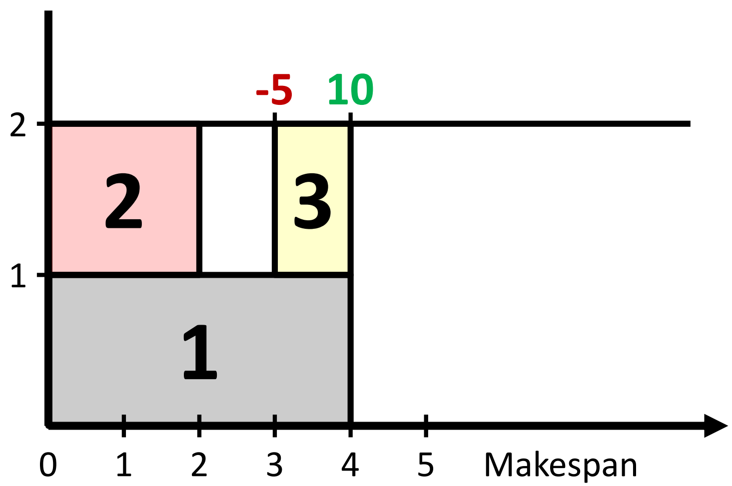
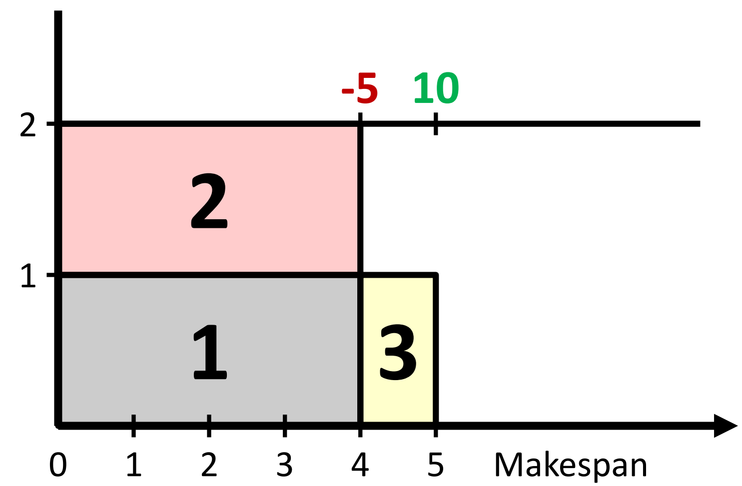


NPV = 3.00

eNPV = 2.86



NPV = 2.71

THE RCPSP

The RCPSP: Facts & figures

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- Still 48 open problems for **J60** (a set of benchmark problems)

The RCPSP: A brief (incomplete) timeline

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1959

Bowman (MIT): first optimal solution

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1959 1983

Blazewicz (Poznan): proof that RCPSP is NP complete

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2000 — Age of heuristics

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1992

1959 1983 1998 2000 2006

Age of MILP solvers

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2000 — Age of heuristics

2006 — Age of MILP solvers

2018 — Creemers (IESEG): new state-of-the-art

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  - We expect final results to be even better

MARKOVIAN PERT NETWORKS: A NEW CTMC

Agenda

- CTMC of Kulkarni and Adlakha (1986)
- New CTMC
- Comparison of performance for the SRCPSP:
  - CPU times
  - Memory requirements
  - New state-of-the-art results
- Comparison of performance for the SNPV:
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- Conclusion

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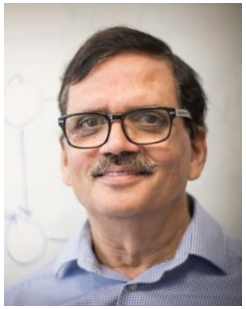

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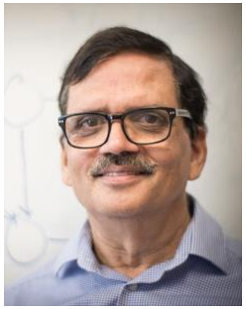

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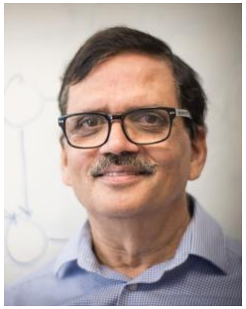

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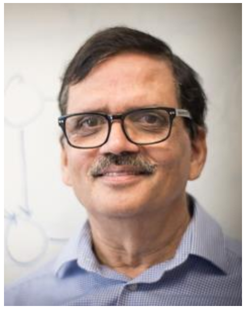

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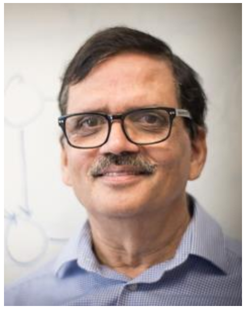

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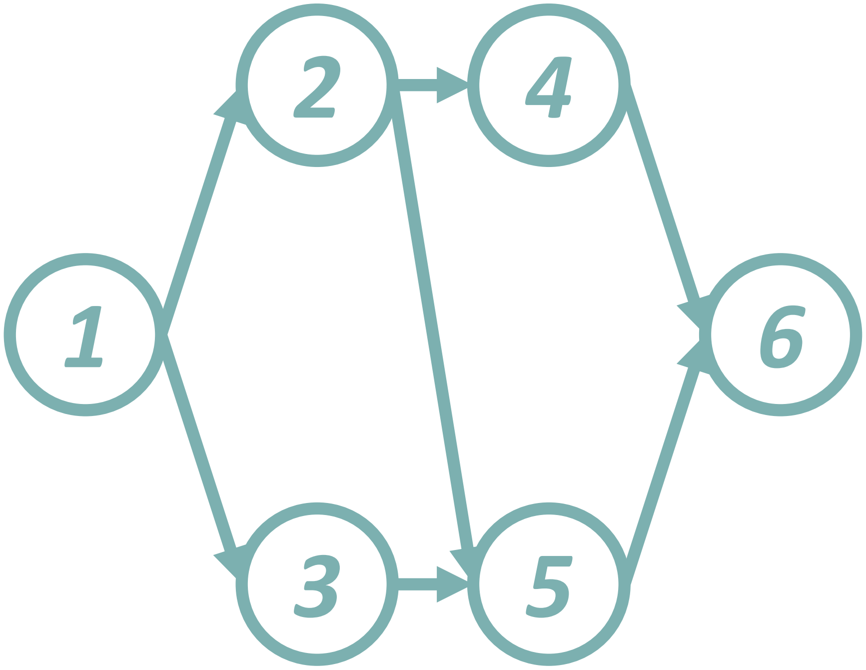
$\Rightarrow$For a project with n activities there are up to 3^n states!

Example: State space



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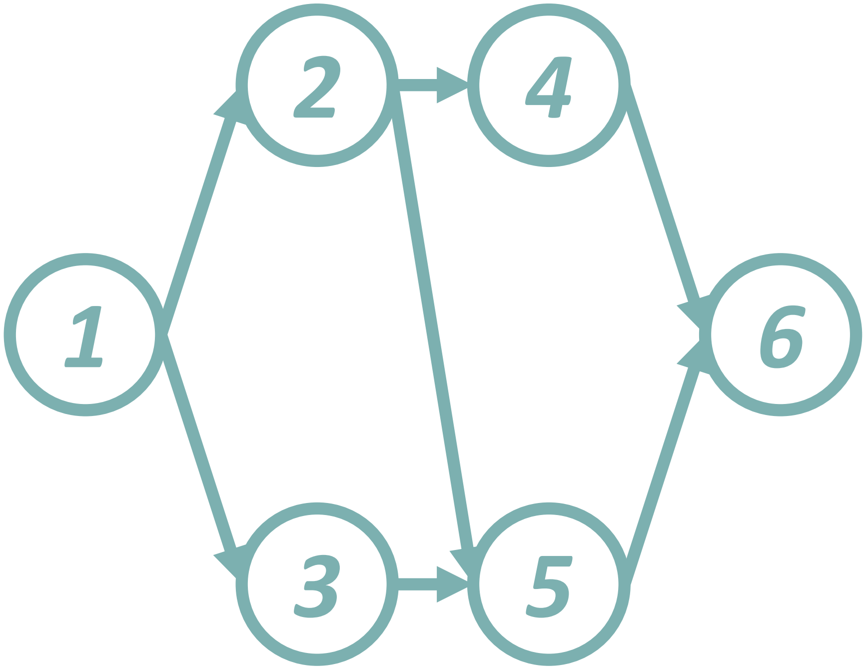

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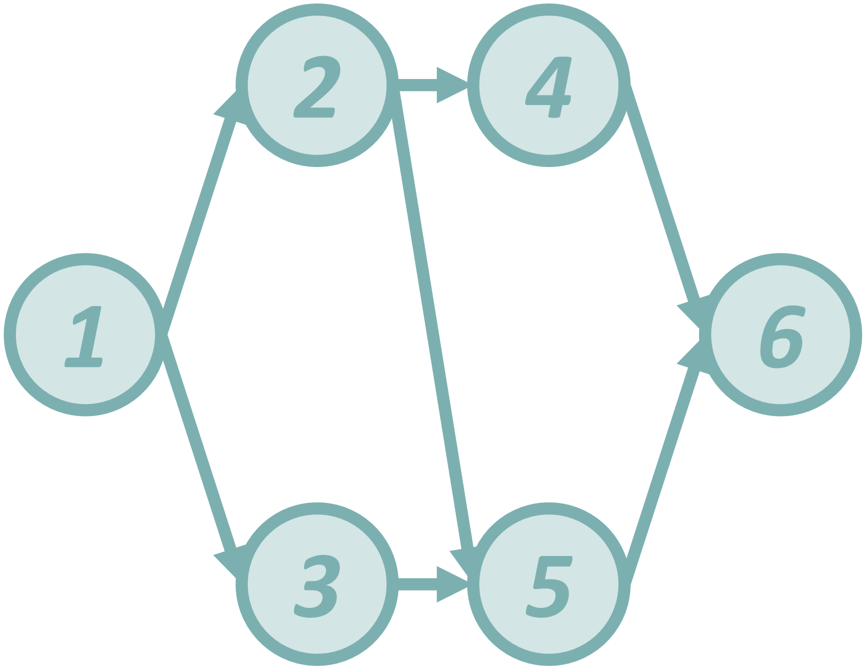

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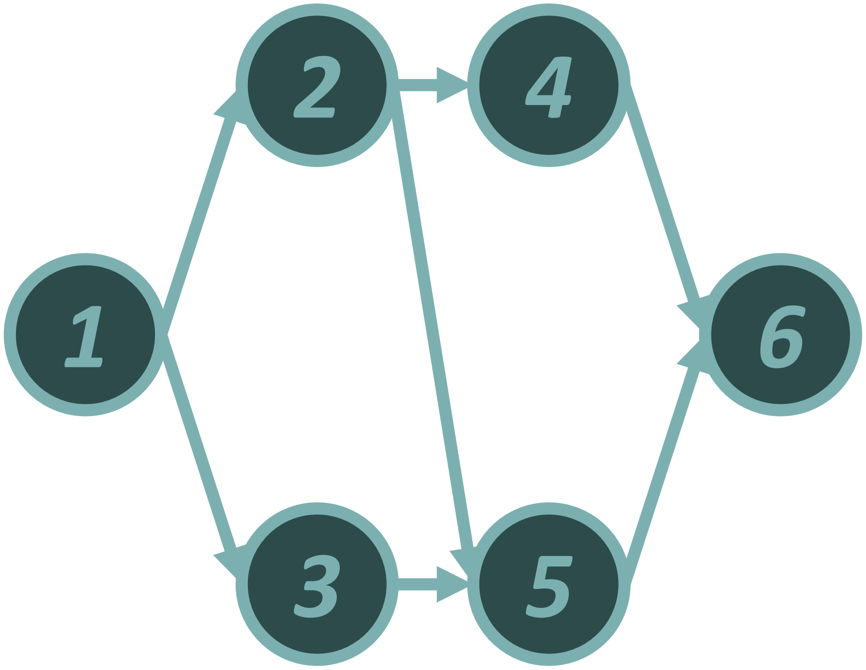

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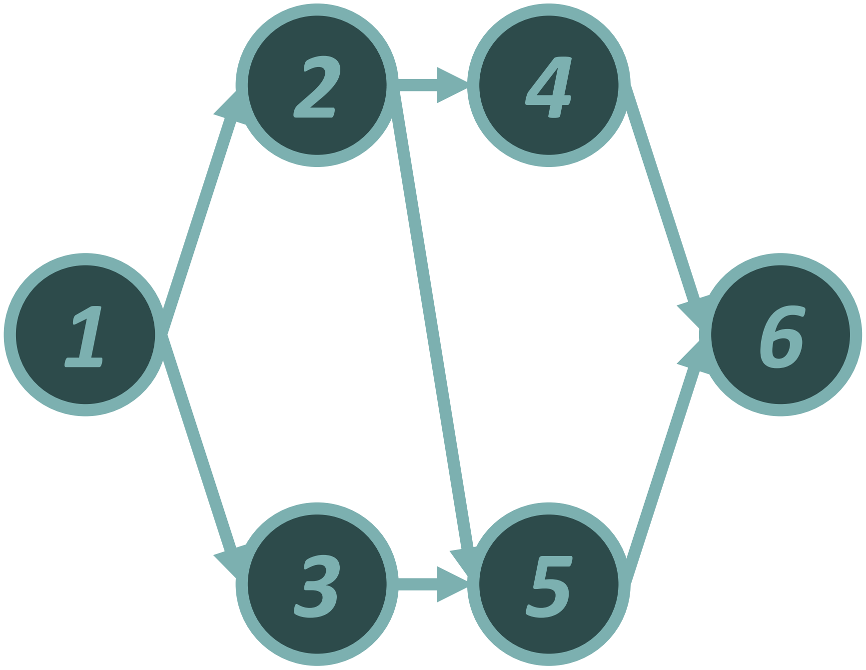

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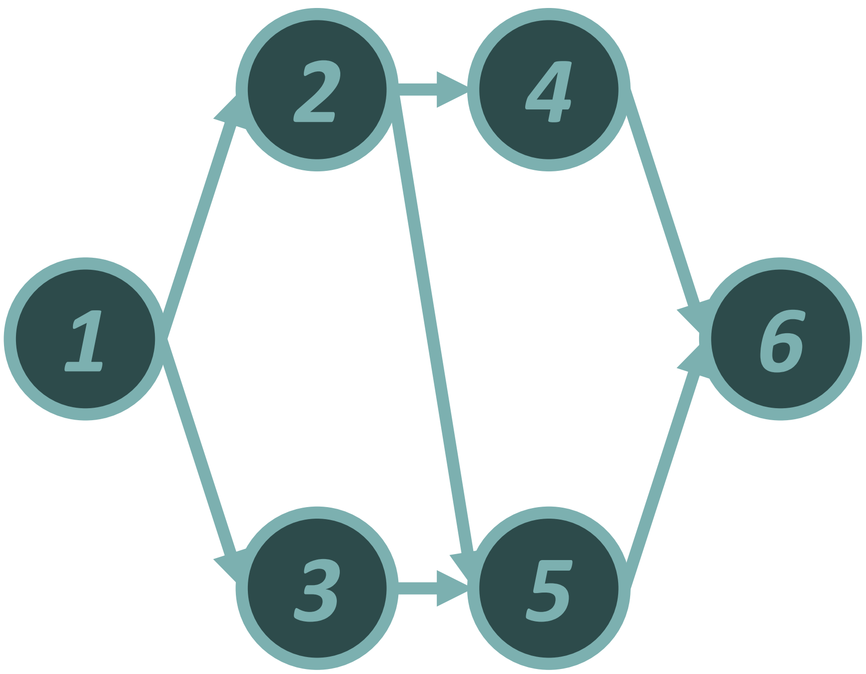

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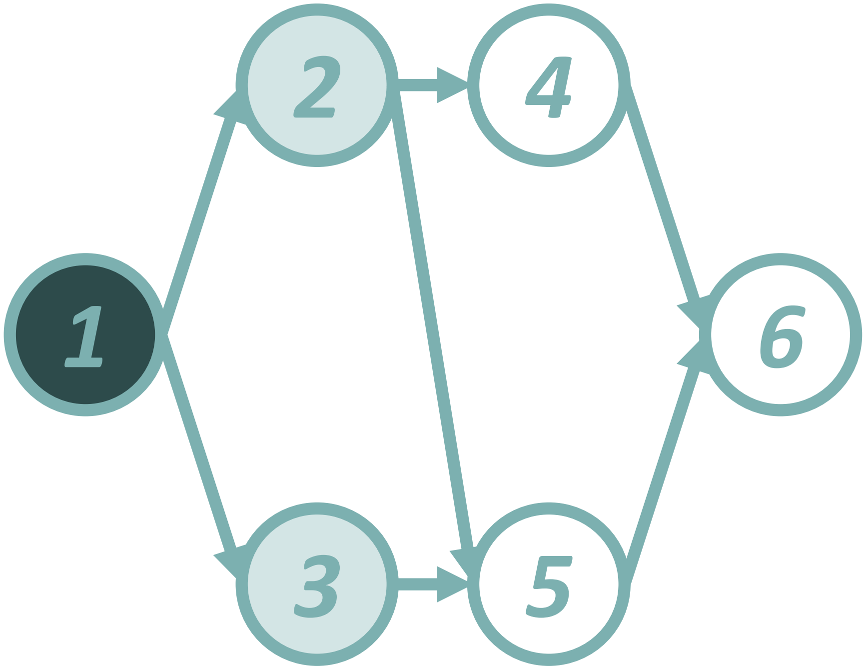

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- Example feasible state:

$\boldsymbol{\theta} = \{2,1,1,0,0,0\}$

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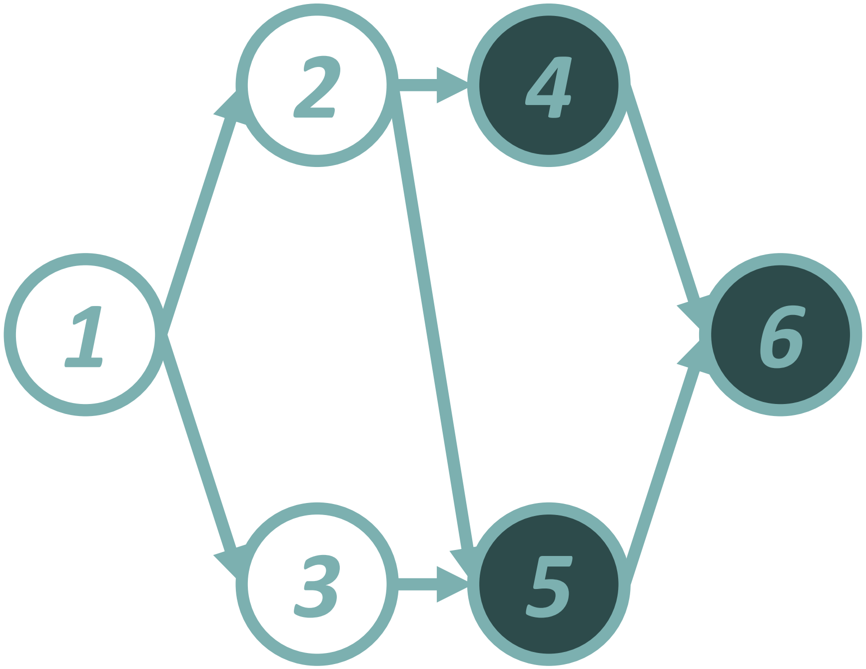

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- The state of the system is represented by a vector:

$\boldsymbol{\theta} = \{\theta_1, \theta_2, \ldots \theta_n\}$

- Up to 3^n = 729 states
- Example feasible state:

$\boldsymbol{\theta} = \{2,1,1,0,0,0\}$

- Example Infeasible state:

$\boldsymbol{\theta} = \{0,0,0,2,2,2\}$

Agenda

- CTMC of Kulkarni and Adlakha (1986)
- New CTMC
- Comparison of performance for the SRCPSP:
  - CPU times
  - Memory requirements
  - New state-of-the-art results
- Comparison of performance for the SNPV:
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  - Memory requirements
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New CTMC

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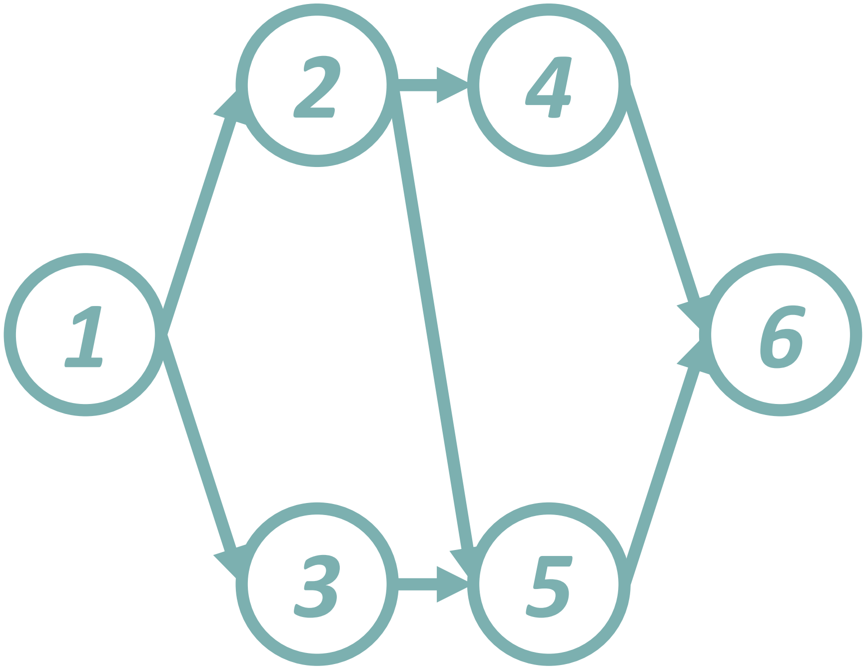
$\Rightarrow$ up to 2^n states (instead of 3^n states)

$\Rightarrow$ Huge reduction in memory requirements (= THE bottleneck for CTMC of Kulkarni & Adlakha)

- A potential "drawback" is that the new CTMC allows activities to be preempted

Example: State space



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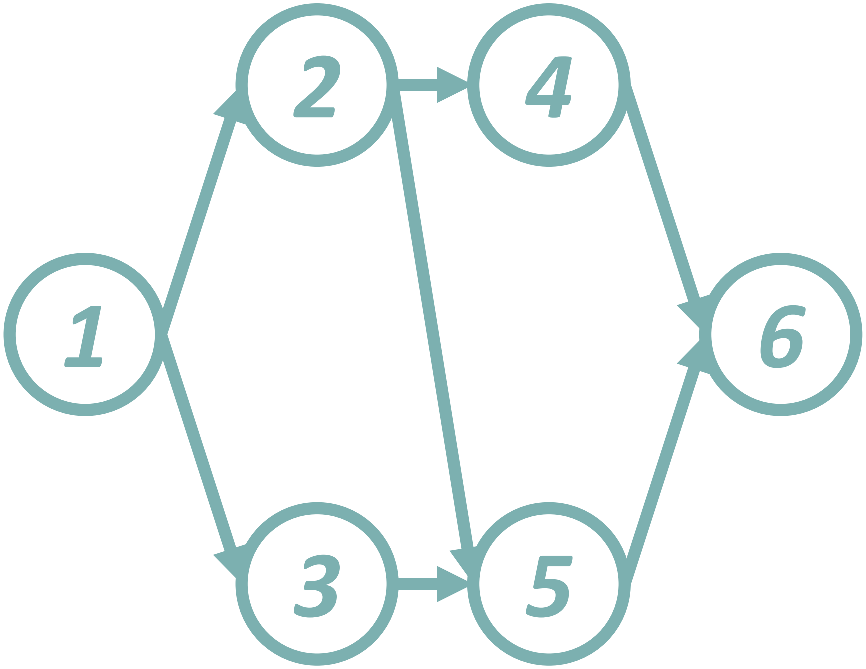
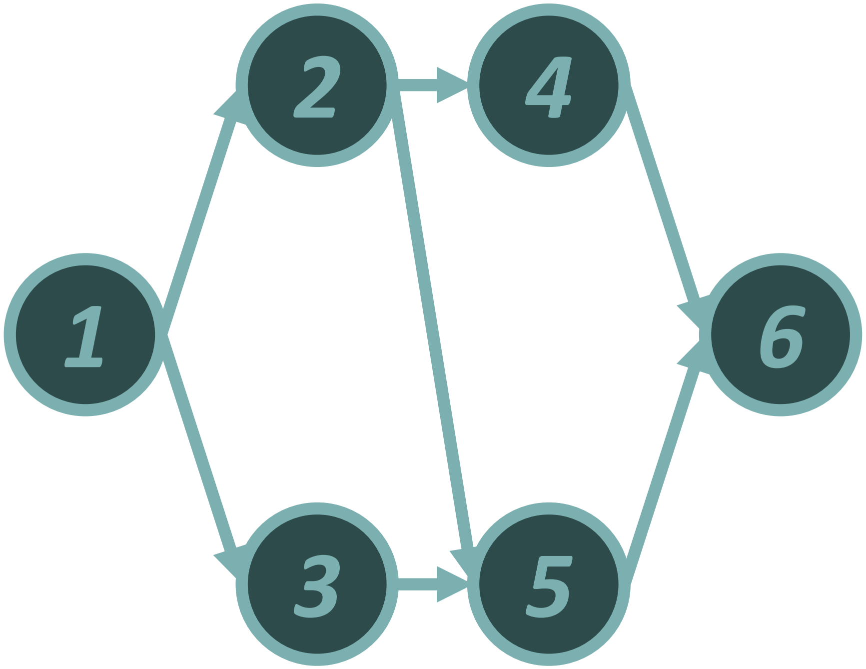


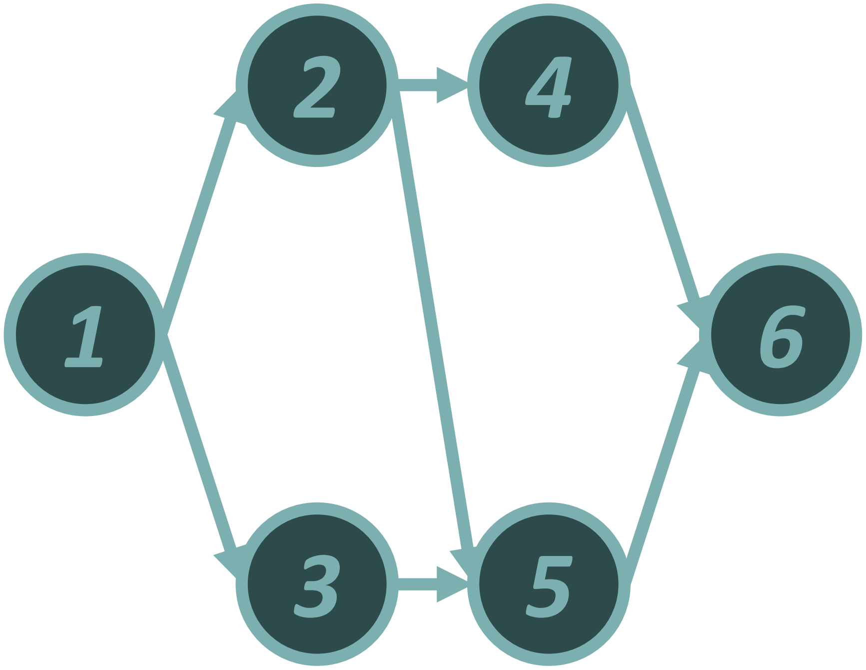
- An activity ***j*** is either:
  - Idle (θ_j=0)

Example: State space



- An activity ***j*** is either:
  - Idle (θ_j=0)
  - Finished (θ_j=1)

Example: State space



- An activity ***j*** is either:
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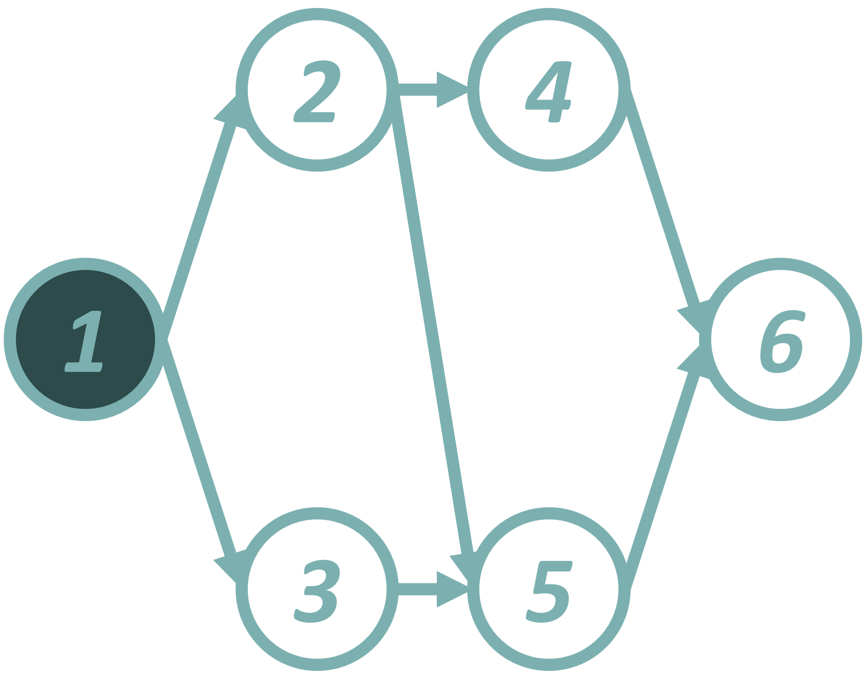
  - Finished (θ_j=1)
- Up to 2^n = 64 states

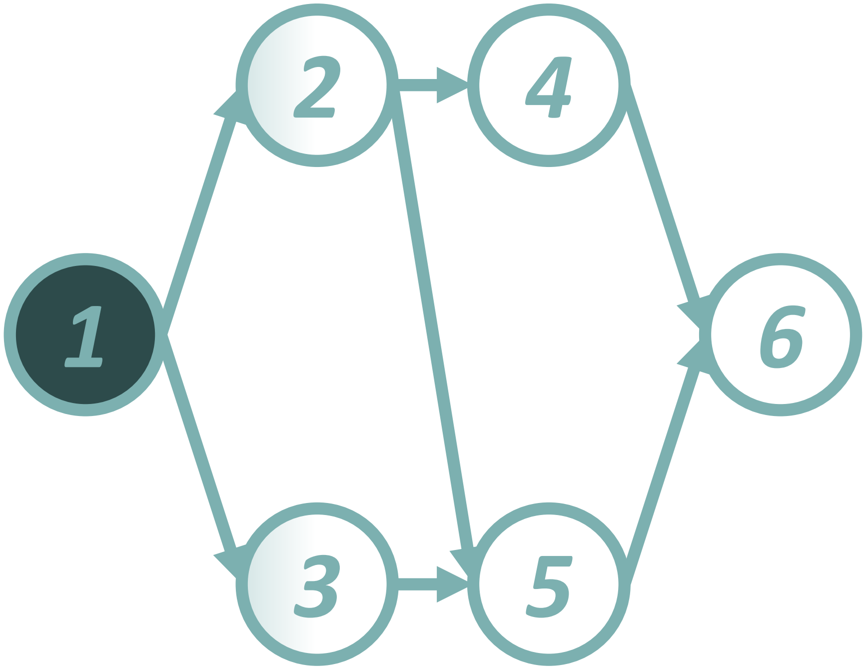
Example: State space



- An activity ***j*** is either:
  - Idle (θ_j=0)
  - Finished (θ_j=1)
- Up to 2^n = 64 states
- Example feasible state:

$\boldsymbol{\theta}$ = {1,0,0,0,0,0}

Example: State space

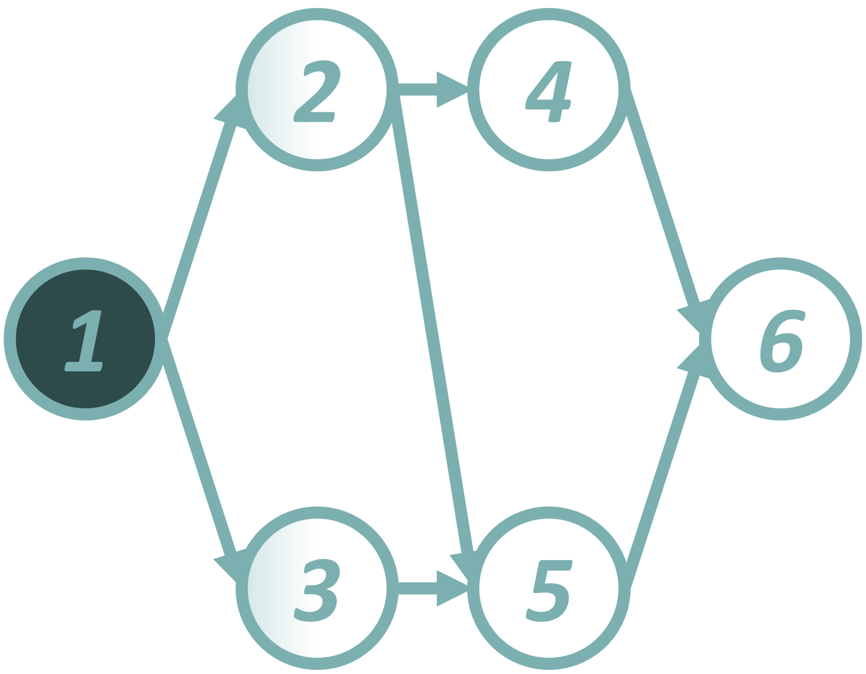


- An activity ***j*** is either:
  - Idle (θ_j=0)
  - Finished (θ_j=1)
- Up to 2^n = 64 states
- Example feasible state:

$\boldsymbol{\theta}$ = *{1,0,0,0,0,0}*

- What activities are ongoing? 2? 3? 2 and 3?

Example: State space



- An activity j is either:
  - Idle (θ_j=0)
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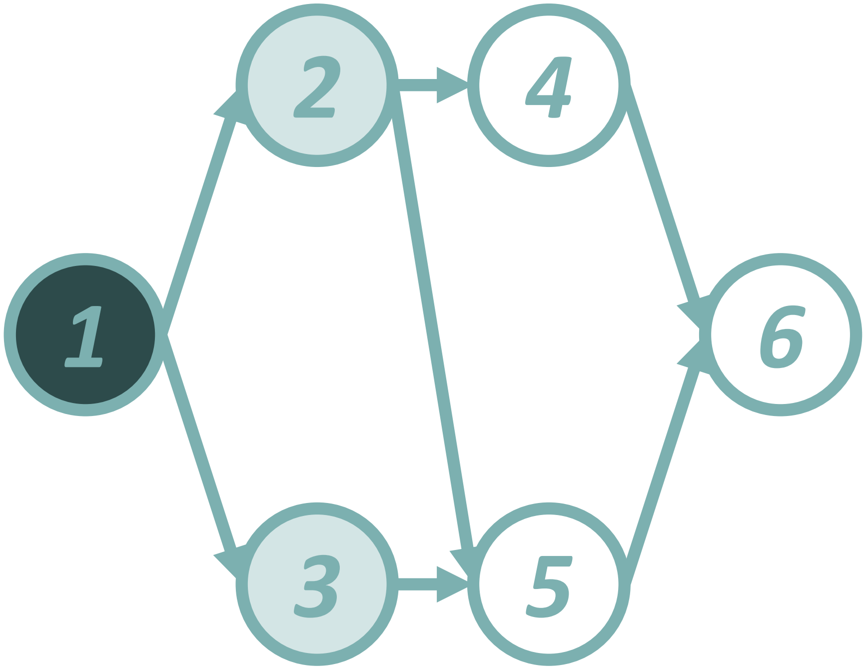
- Up to 2^n = 64 states
- Example feasible state:

$\boldsymbol{\theta}$ = $\{1,0,0,0,0,0\}$

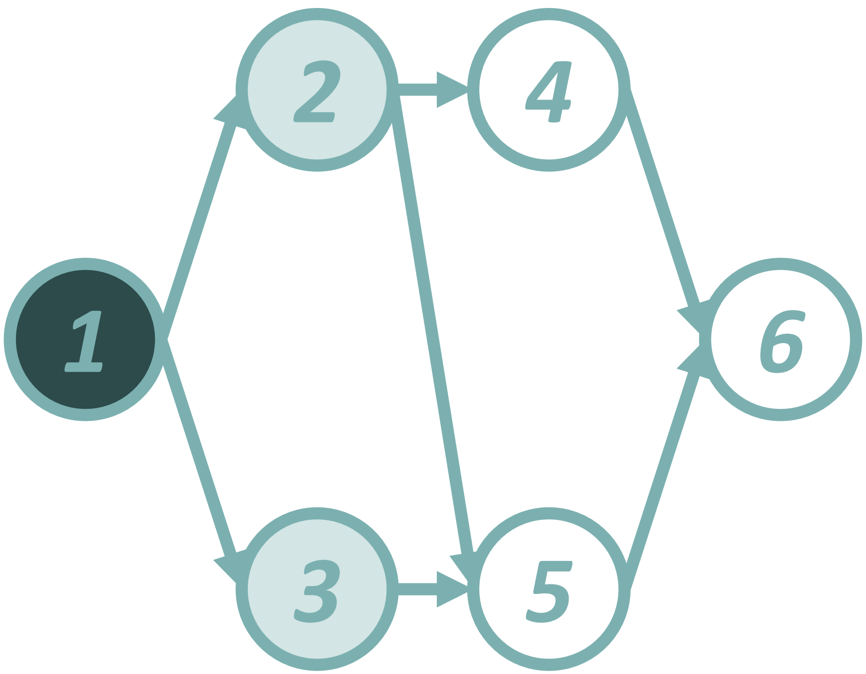
- What activities are ongoing? 2? 3? 2 and 3?
- Preemption is possible

Example: State space



In this state, it is optimal if activities 2 & 3 are ongoing

Example: State space



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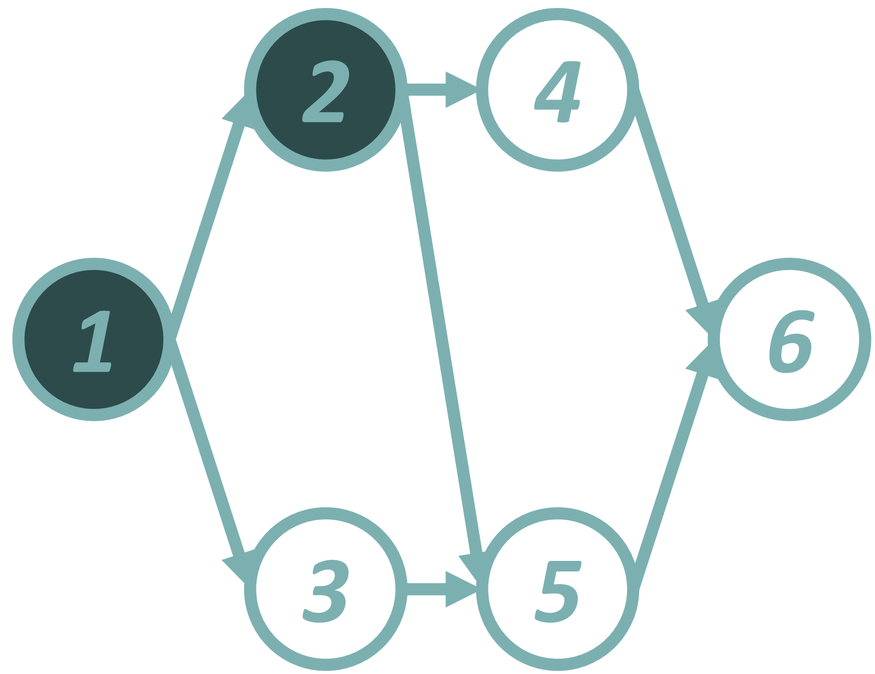
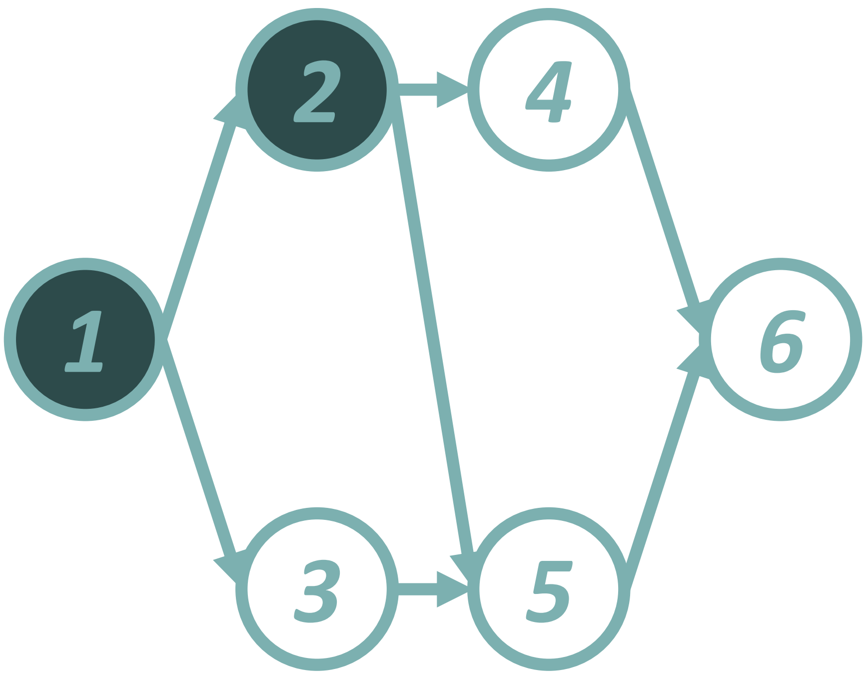


Activity 2 finishes ➔ we end up in state θ = *{1,1,0,0,0,0}*

Example: State space



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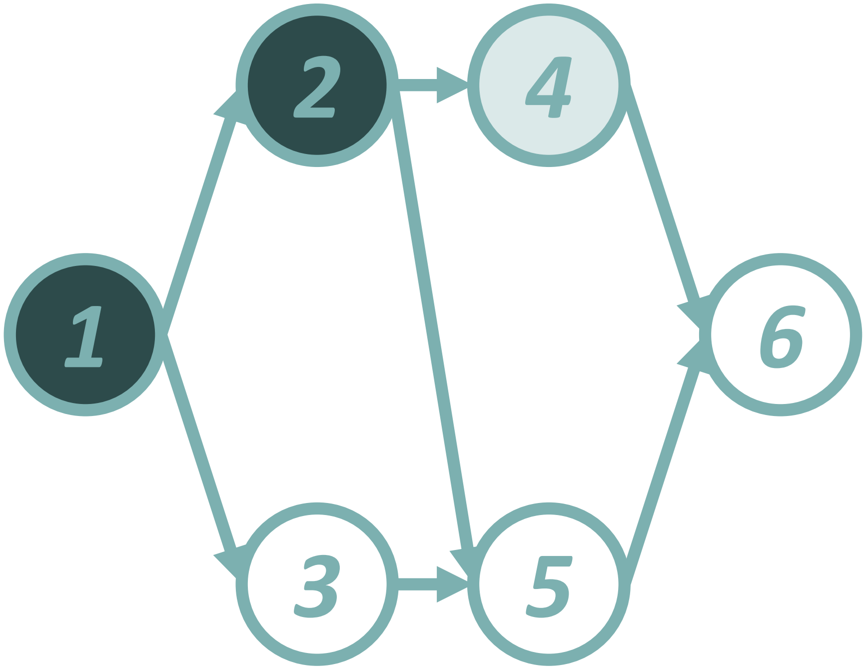
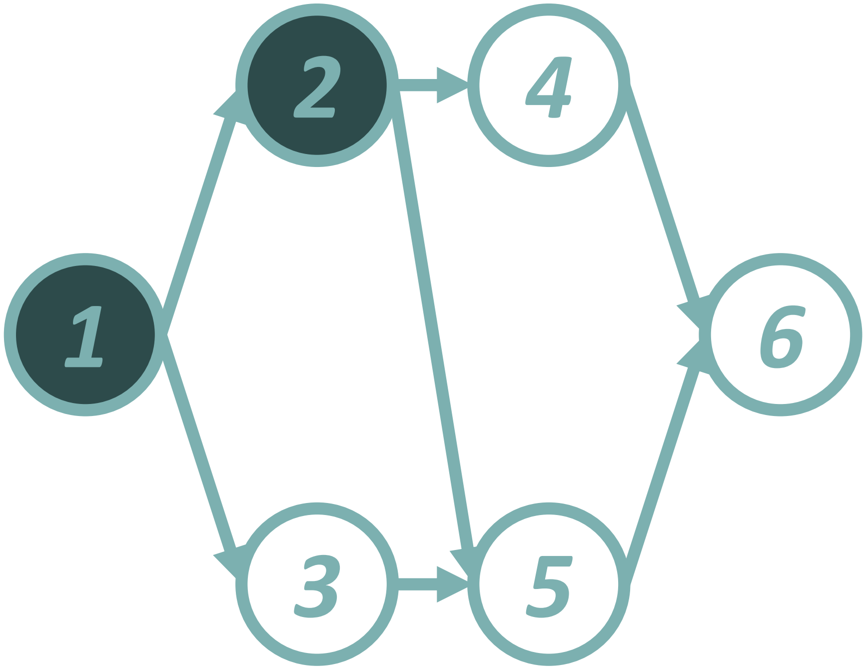
Example: State space





Here, it is optimal if activity 4 is ongoing ➔activity 3 is preempted!

Activity 2 finishes ➔we end up in state θ = {1,1,0,0,0,0}

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Creemers (2015)

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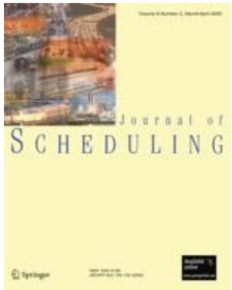

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- Bottleneck = memory requirements

SRCPSP

2015 (JOS) Instances Solved

OLD CTMC	
Instances solved (out of 480)	
J30	480
J60	303
J90	NA
J120	NA

SRCPSP

2015 (JOS) CPU Times

OLD CTMC	
Instances solved (out of 480)	
J30	480
J60	303
J90	NA
J120	NA

OLD CTMC	
Average CPU time (s)	
J30	0.48
J60	1591
J90	NA
J120	NA

SRCPSP

2015 (JOS) VS new CTMC

NEW CTMC	
Avg CPU time (s) for same inst.	
J30	0.02
J60	81.6
J90	NA
J120	NA

OLD CTMC	
Average CPU time (s)	
J30	0.48
J60	1591
J90	NA
J120	NA

SRCPSP

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J30	0.02
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J90	NA
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OLD CTMC	
Average CPU time (s)	
J30	0.48
J60	1591
J90	NA
J120	NA

On average, we improve computation times by a factor of 19!

SRCPSP

2015 (JOS) Memory Requirements

OLD CTMC	
Instances solved (out of 480)	
J30	480
J60	303
J90	NA
J120	NA

SRCPSP

2015 (JOS) Memory Requirements

OLD CTMC	
Instances solved (out of 480)	
J30	480
J60	303
J90	NA
J120	NA

OLD CTMC	
Average max # states (x1000)	
J30	176
J60	374499
J90	NA
J120	NA

SRCPSP

2015 (JOS) VS new CTMC

NEW CTMC	
Avg max # states (x1K) for = inst.	
J30	1.99
J60	508
J90	NA
J120	NA

OLD CTMC	
Average max # states (x1000)	
J30	176
J60	374499
J90	NA
J120	NA

SRCPSP

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NEW CTMC	
Avg max # states (x1K) for = inst.	
J30	1.99
J60	508
J90	NA
J120	NA

OLD CTMC	
Average max # states (x1000)	
J30	176
J60	374499
J90	NA
J120	NA

On average, we reduce memory requirements by a factor of 733!

SRCPSP

New CTMC Instances Solved

NEW CTMC	
Instances solved (out of 480)	
J30	480
J60	480
J90	196
J120	10

SRCPSP

New CTMC Instances Solved

NEW CTMC	
Instances solved (out of 480)	
J30	480
J60	480
J90	196
J120	10

We are the first to solve instances of the J90 and J120 data sets to optimality!

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- Bottleneck = memory requirements

SNPV

2010 (ORL) Instances Solved

OLD CTMC			
Instances solved (out of 30)			
	OS = 0.8	OS = 0.6	OS = 0.4
n = 10	30	30	30
n = 20	30	30	30
n = 30	30	30	30
n = 40	30	30	29
n = 50	30	30	4
n = 60	30	30	0
n = 70	30	22	0

SNPV

2010 (ORL) CPU Times

OLD CTMC			
Instances solved (out of 30)			
	OS = 0.8	OS = 0.6	OS = 0.4
n = 10	30	30	30
n = 20	30	30	30
n = 30	30	30	30
n = 40	30	30	29
n = 50	30	30	4
n = 60	30	30	0
n = 70	30	22	0

OLD CTMC			
Average CPU time (s)			
	OS = 0.8	OS = 0.6	OS = 0.4
n = 10	0	0	0
n = 20	0	0	0
n = 30	0	0	27
n = 40	0	7	2338
n = 50	0	100	52268
n = 60	1	2210	NA
n = 70	3	17496	NA

SNPV

2010 (ORL) VS new CTMC

NEW CTMC			
Average CPU time (s) for same instances			
	OS = 0.8	OS = 0.6	OS = 0.4
n = 10	0	0	0
n = 20	0	0	0
n = 30	0	0	0
n = 40	0	0	7
n = 50	0	1	82
n = 60	0	6	NA
n = 70	0	34	NA

OLD CTMC			
Average CPU time (s)			
	OS = 0.8	OS = 0.6	OS = 0.4
n = 10	0	0	0
n = 20	0	0	0
n = 30	0	0	27
n = 40	0	7	2338
n = 50	0	100	52268
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n = 40	0	7	2338
n = 50	0	100	52268
n = 60	1	2210	NA
n = 70	3	17496	NA

On average, we improve computation times by a factor of 492!

SNPV

2010 (ORL) Memory Requirements

OLD CTMC			
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	OS = 0.8	OS = 0.6	OS = 0.4
n = 10	30	30	30
n = 20	30	30	30
n = 30	30	30	30
n = 40	30	30	29
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SNPV

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n = 30	30	30	30
n = 40	30	30	29
n = 50	30	30	4
n = 60	30	30	0
n = 70	30	22	0

OLD CTMC			
Average max # states (x1000)			
	OS = 0.8	OS = 0.6	OS = 0.4
n = 10	0	0	1
n = 20	0	4	55
n = 30	2	49	1560
n = 40	8	534	47073
n = 50	27	4346	526020
n = 60	92	42279	NA
n = 70	287	216028	NA

SNPV

2010 (ORL) VS new CTMC

NEW CTMC			
Avg max # states (x1000) for same inst.			
	OS = 0.8	OS = 0.6	OS = 0.4
n = 10	0	0	0
n = 20	0	0	2
n = 30	0	2	17
n = 40	1	9	172
n = 50	2	40	1055
n = 60	4	175	NA
n = 70	8	593	NA

OLD CTMC			
Average max # states (x1000)			
	OS = 0.8	OS = 0.6	OS = 0.4
n = 10	0	0	1
n = 20	0	4	55
n = 30	2	49	1560
n = 40	8	534	47073
n = 50	27	4346	526020
n = 60	92	42279	NA
n = 70	287	216028	NA

SNPV

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Avg max # states (x1000) for same inst.			
	OS = 0.8	OS = 0.6	OS = 0.4
n = 10	0	0	0
n = 20	0	0	2
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n = 60	4	175	NA
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n = 30	2	49	1560
n = 40	8	534	47073
n = 50	27	4346	526020
n = 60	92	42279	NA
n = 70	287	216028	NA

On average, we reduce memory requirements by a factor of 403!

SNPV

New CTMC Instances Solved

NEW CTMC			
Instances solved (out of 30)			
	OS = 0.8	OS = 0.6	OS = 0.4
n = 10	30	30	30
n = 20	30	30	30
n = 30	30	30	30
n = 40	30	30	30
n = 50	30	30	30
n = 60	30	30	30
n = 70	30	30	30

SNPV

New CTMC CPU Times

NEW CTMC			
Instances solved (out of 30)			
	OS = 0.8	OS = 0.6	OS = 0.4
n = 10	30	30	30
n = 20	30	30	30
n = 30	30	30	30
n = 40	30	30	30
n = 50	30	30	30
n = 60	30	30	30
n = 70	30	30	30

NEW CTMC			
Average CPU time (s)			
	OS = 0.8	OS = 0.6	OS = 0.4
n = 10	0	0	0
n = 20	0	0	0
n = 30	0	0	0
n = 40	0	0	22
n = 50	0	1	476
n = 60	0	11	16869
n = 70	0	99	263012

SNPV

New CTMC CPU Times

NEW CTMC			
Instances solved (out of 30)			
	OS = 0.8	OS = 0.6	OS = 0.4
n = 10	30	30	30
n = 20	30	30	30
n = 30	30	30	30
n = 40	30	30	30
n = 50	30	30	30
n = 60	30	30	30
n = 70	30	30	30

NEW CTMC			
Average CPU time (s)			
	OS = 0.8	OS = 0.6	OS = 0.4
n = 10	0	0	0
n = 20	0	0	0
n = 30	0	0	0
n = 40	0	0	22
n = 50	0	1	476
n = 60	0	11	16869
n = 70	0	99	263012

CPU times have become the new bottleneck

SNPV

To preempt or not to preempt?

SNPV

To preempt or not to preempt?

- If an activity has a zero cost, it is optimal to start that activity as early as possible

SNPV

To preempt or not to preempt?

- If an activity has a zero cost, it is optimal to start that activity as early as possible
- If at time t activity i is preempted, the remainder of activity i joins the set of eligible activities

SNPV

To preempt or not to preempt?

- If an activity has a zero cost, it is optimal to start that activity as early as possible
- If at time *t* activity *i* is preempted, the remainder of activity *i* joins the set of eligible activities
- The remainder of activity *i* has a zero cost (the cost has already been incurred at the start of activity *i*)

SNPV

To preempt or not to preempt?

- If an activity has a zero cost, it is optimal to start that activity as early as possible
- If at time t activity i is preempted, the remainder of activity i joins the set of eligible activities
- The remainder of activity i has a zero cost (the cost has already been incurred at the start of activity i)

$\Rightarrow$ It is optimal to start the remainder of activity i at time t

SNPV

To preempt or not to preempt?

- If an activity has a zero cost, it is optimal to start that activity as early as possible
- If at time t activity i is preempted, the remainder of activity i joins the set of eligible activities
- The remainder of activity i has a zero cost (the cost has already been incurred at the start of activity i)

$\Rightarrow$ It is optimal to start the remainder of activity i at time t

$\Rightarrow$ It is optimal not to preempt activity i

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- New CTMC that only keeps track of finished activities

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- Significantly reduces memory requirements when compared with CTMC of Kulkarni & Adlakha

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Conclusion

- New CTMC that only keeps track of finished activities
- Significantly reduces memory requirements when compared with CTMC of Kulkarni & Adlakha
- New state-of-the-art for solving the SRCPSP and the SNPV
- Bottleneck shifted from memory requirements to CPU times

Conclusion

- New CTMC that only keeps track of finished activities
- Significantly reduces memory requirements when compared with CTMC of Kulkarni & Adlakha
- New state-of-the-art for solving the SRCPSP and the SNPV
- Bottleneck shifted from memory requirements to CPU times
- Only "drawback" is that the new CTMC allows activities to be preempted

Conclusion

- New CTMC that only keeps track of finished activities
- Significantly reduces memory requirements when compared with CTMC of Kulkarni & Adlakha
- New state-of-the-art for solving the SRCPSP and the SNPV
- Bottleneck shifted from memory requirements to CPU times
- Only "drawback" is that the new CTMC allows activities to be preempted
- We prove that there is no preemption when solving the SNPV

MOMENTS & DISTRIBUTION OF PROJECT NPV

Agenda

- Introduction
- Serial projects:
  - Single cash flow after a single stage
  - Single cash flow after multiple stages
  - NPV of a serial project
  - Optimal sequence of stages
- General projects
- Conclusions

Agenda

- Introduction
- Serial projects:
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  - Single cash flow after multiple stages
  - NPV of a serial project
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Introduction

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- We study the NPV of a project where:
  - Activities have general duration distributions
  - Cash flows are incurred during the lifetime of the project

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  - Cash flows are incurred during the lifetime of the project
- For such settings, most of the literature has focused on determining the expected NPV (eNPV) of a project

Introduction

- We study the NPV of a project where:
  - Activities have general duration distributions
  - Cash flows are incurred during the lifetime of the project
- For such settings, most of the literature has focused on determining the expected NPV (eNPV) of a project
- Higher moments/distribution of project NPV are currently determined using Monte Carlo simulation

Introduction

- We study the NPV of a project where:
  - Activities have general duration distributions
  - Cash flows are incurred during the lifetime of the project
- For such settings, most of the literature has focused on determining the expected NPV (eNPV) of a project
- Higher moments/distribution of project NPV are currently determined using Monte Carlo simulation
- We develop exact, closed-form expressions for the moments of project NPV & develop an accurate approximation of the NPV distribution itself

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  - Single cash flow after multiple stages
  - NPV of a serial project
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NPV of a single cash flow obtained after a single stage

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- v_w = NPV of cash flow c_w

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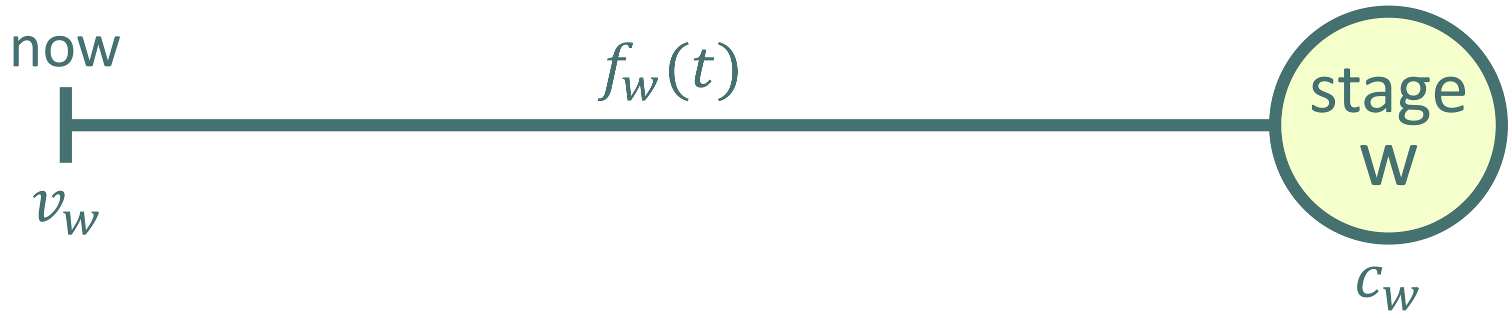

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- $f_w(t)$ = distribution of time until cash flow c_w is incurred

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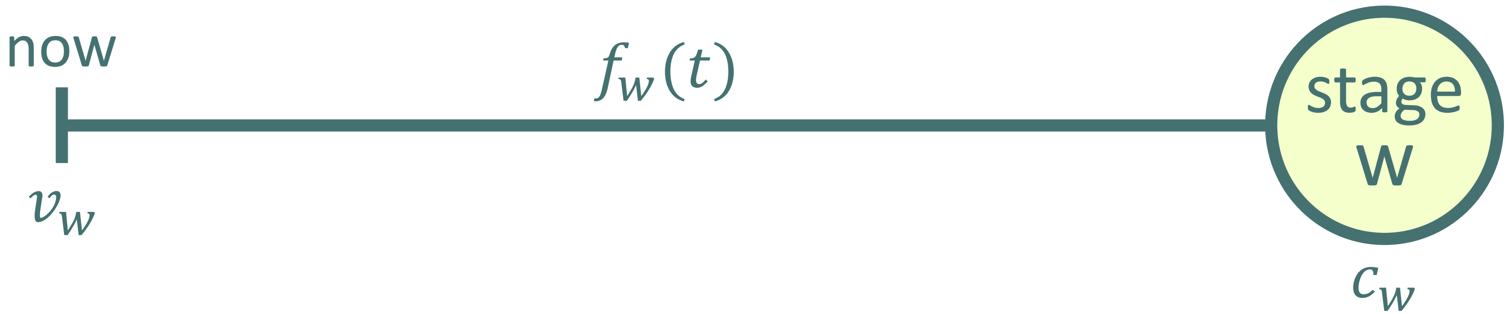

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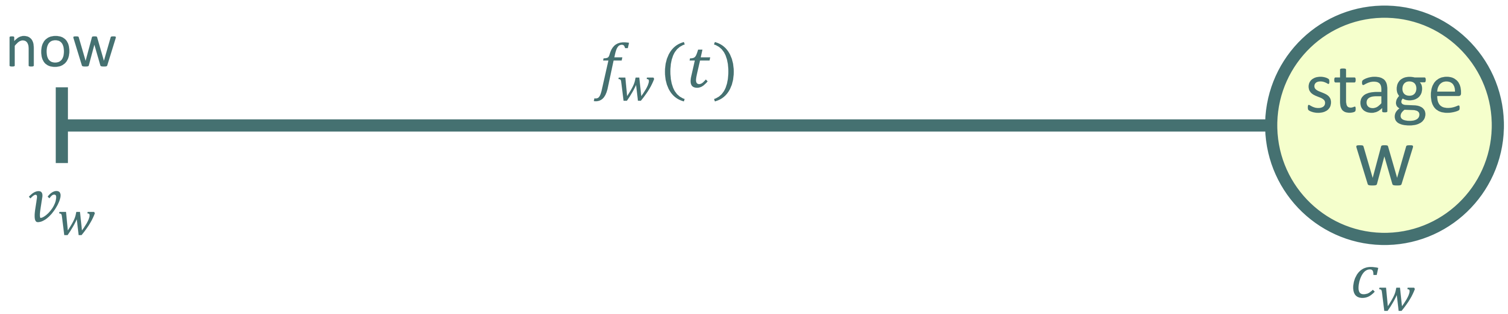

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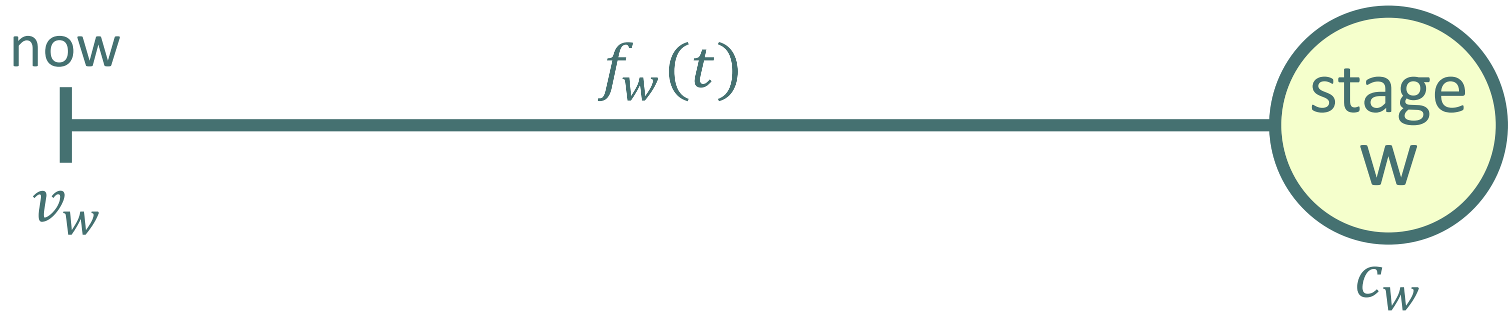

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- $\phi_w(r)$ = discount factor for stage w

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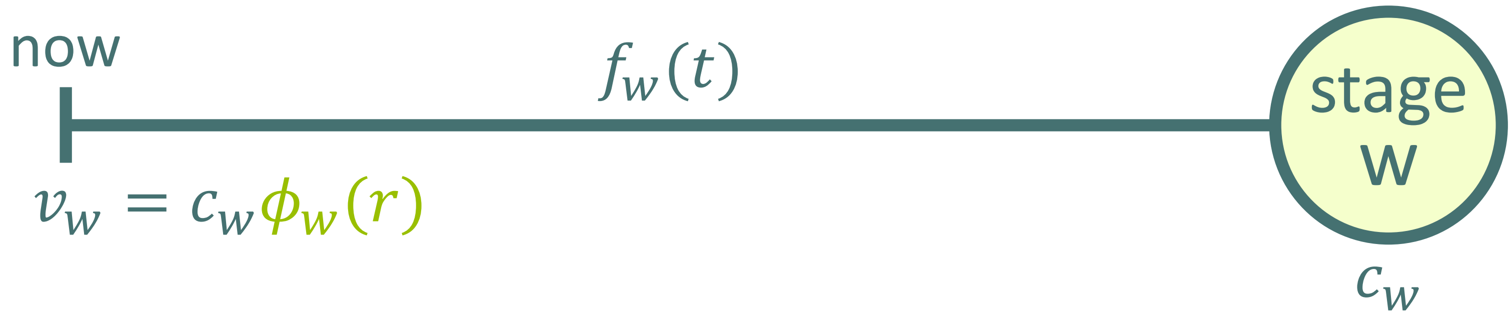

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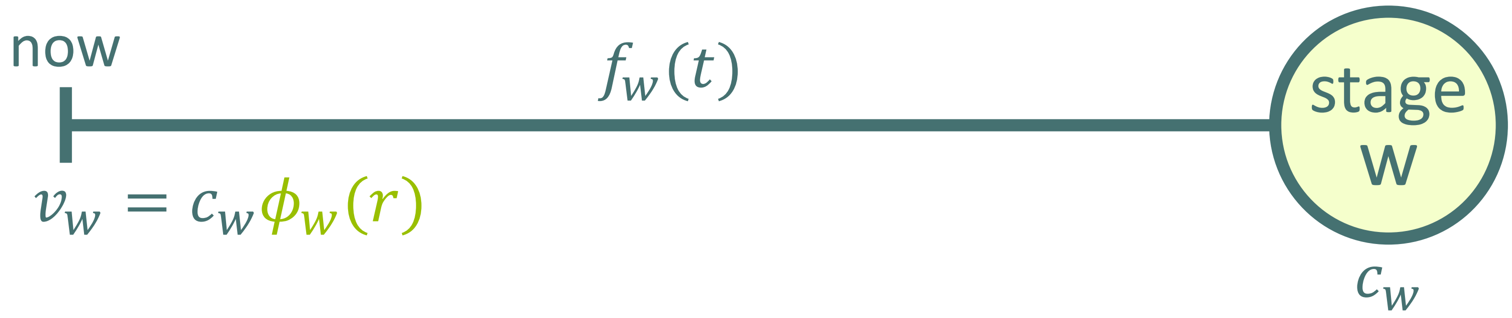

- Using discount factor $\phi_w(r)$, we can obtain the moments of the NPV:
  - $\mu_w = c_w \phi_w(r)$
  - $\sigma_w^2 = c_w^2(\phi_w(2r) - \phi_w^2(r))$
  - $\gamma_w = c_w^3(\phi_w(3r) - 3\phi_w(2r)\phi_w(r) + 2\phi_w^3(r))\, \sigma_w^{-3}$
  - $\theta_w = c_w^4(\phi_w(4r) - 4\phi_w(3r)\phi_w(r) + 6\phi_w(2r)\phi_w^2(r) - 3\phi_w^4(r))\, \sigma_w^{-4}$

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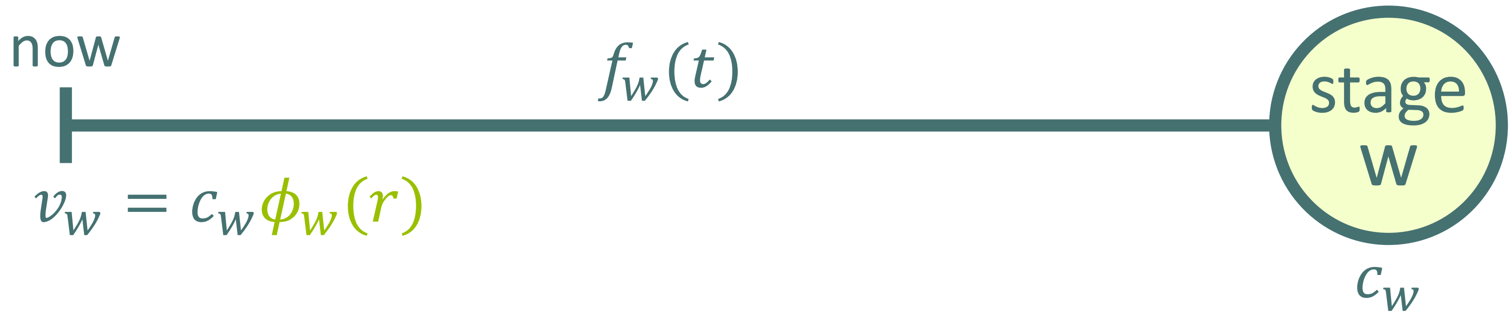

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- The CDF & PDF of the NPV of c_w are:
  - $G_w(v) = 1 - F_w\left(\ln\left(\frac{c_w}{v}\right) r^{-1}\right)$
  - $g_w(v) = \frac{f_w\left(\ln\left(\frac{c_w}{v}\right) r^{-1}\right)}{|r| v}$

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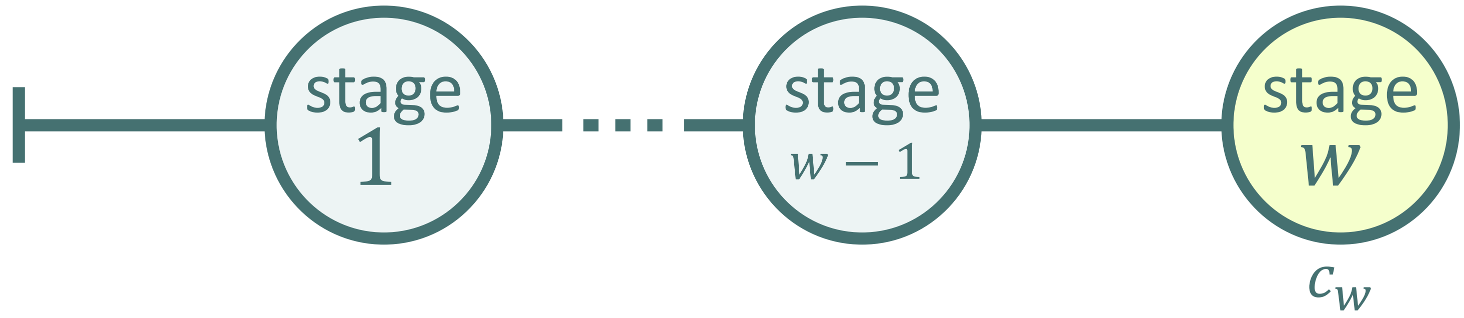

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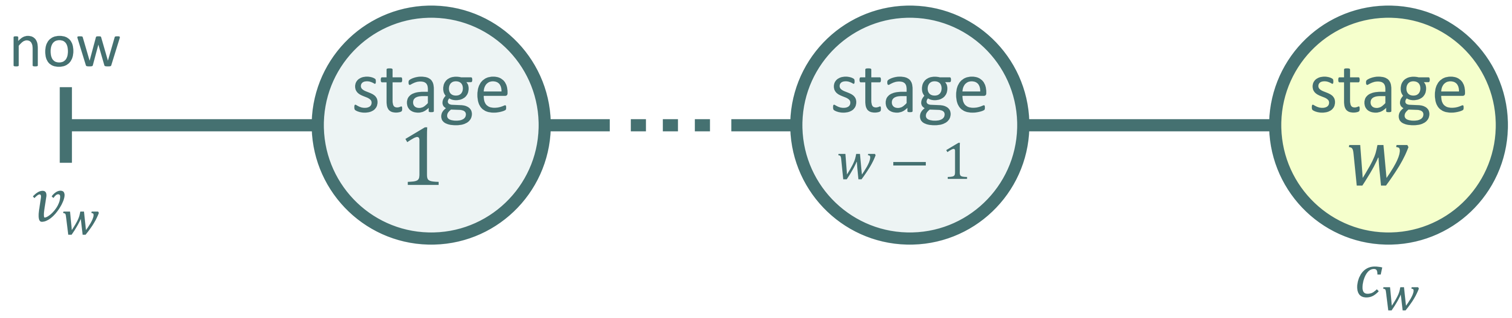

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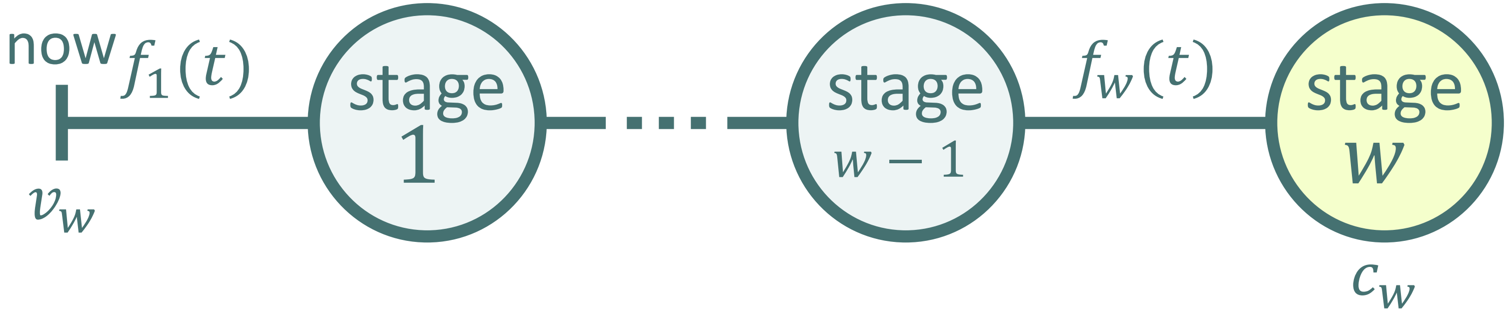

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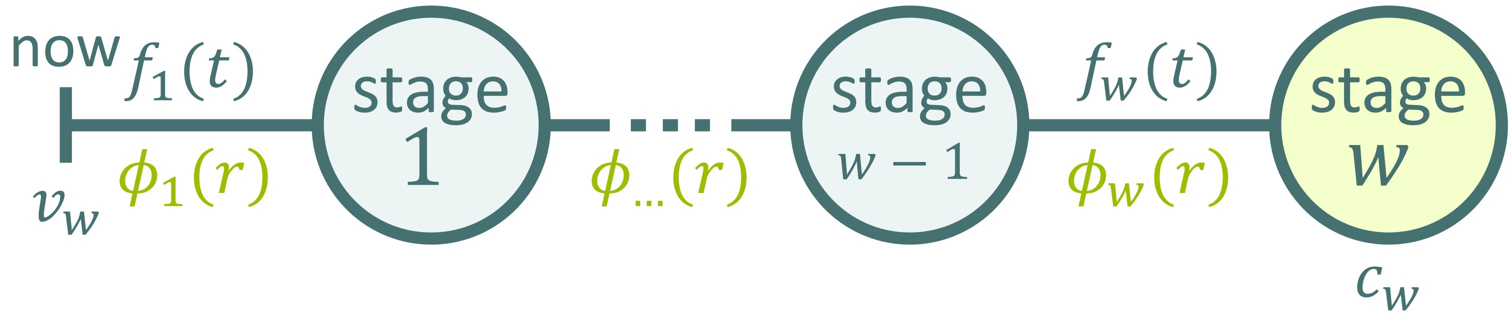

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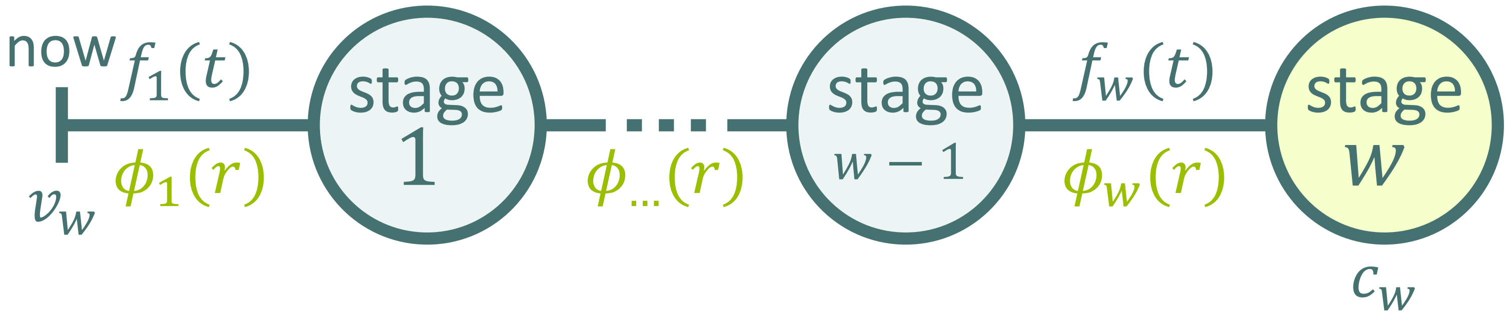

$$v_w = c_w \phi_1(r) \dots \phi_w(r)$$

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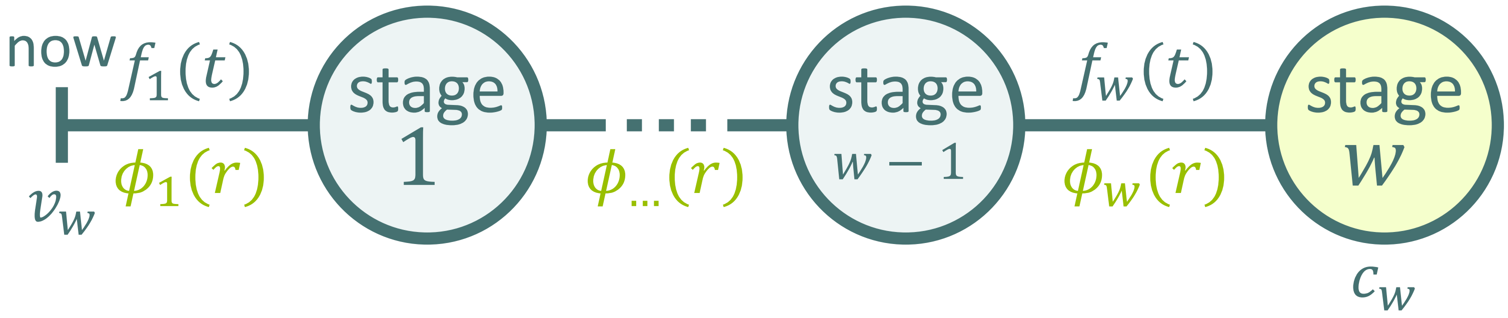

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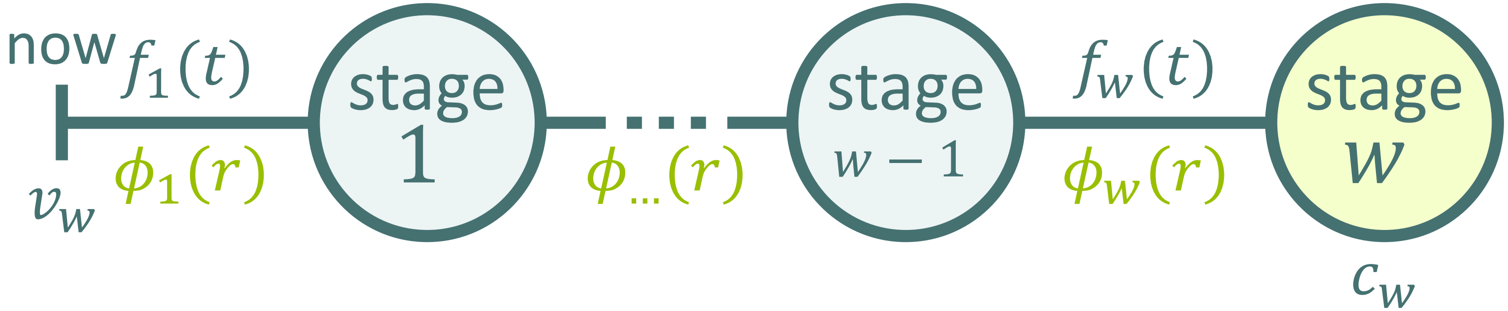

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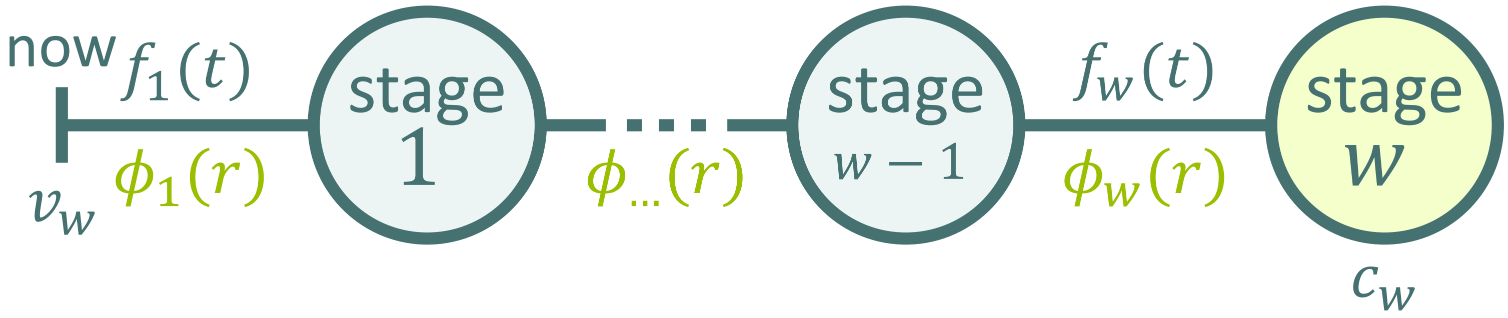

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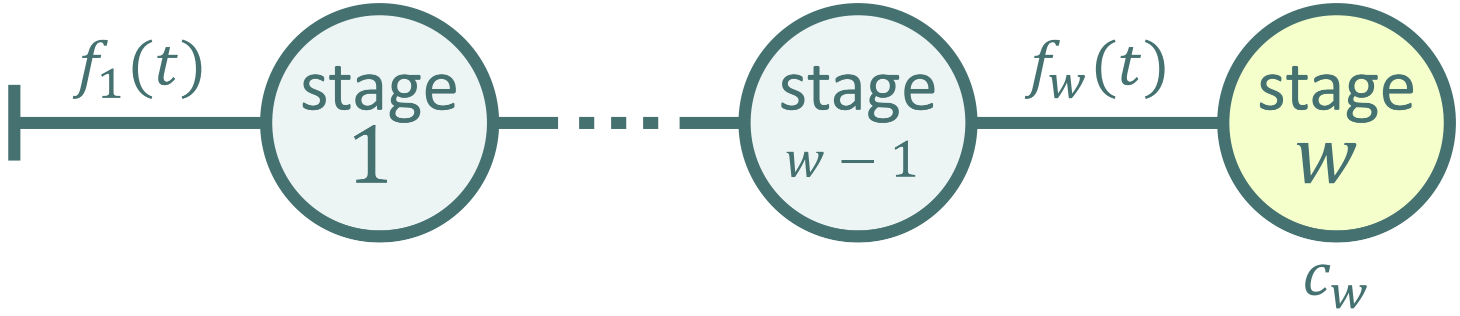

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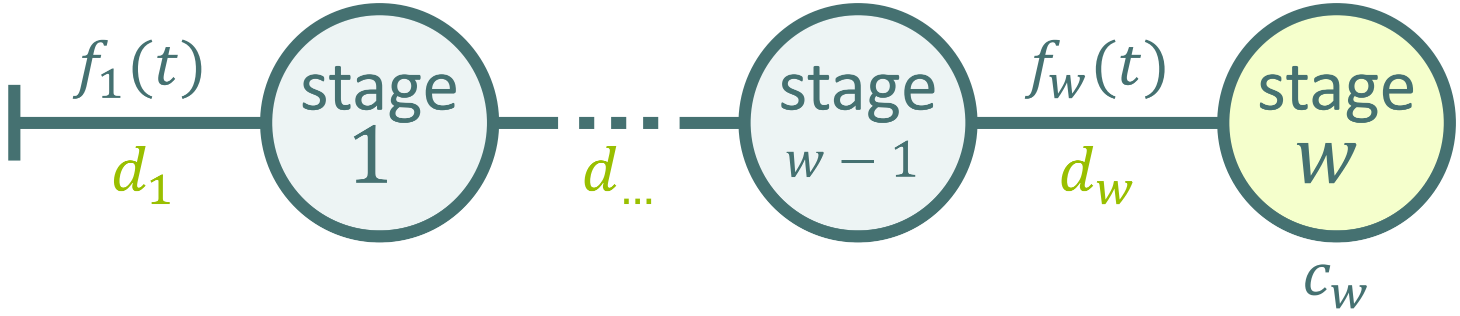

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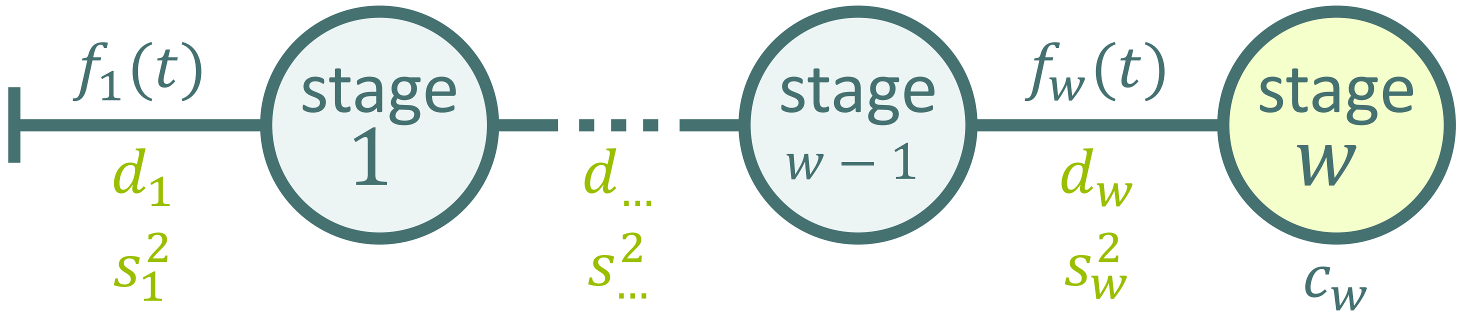

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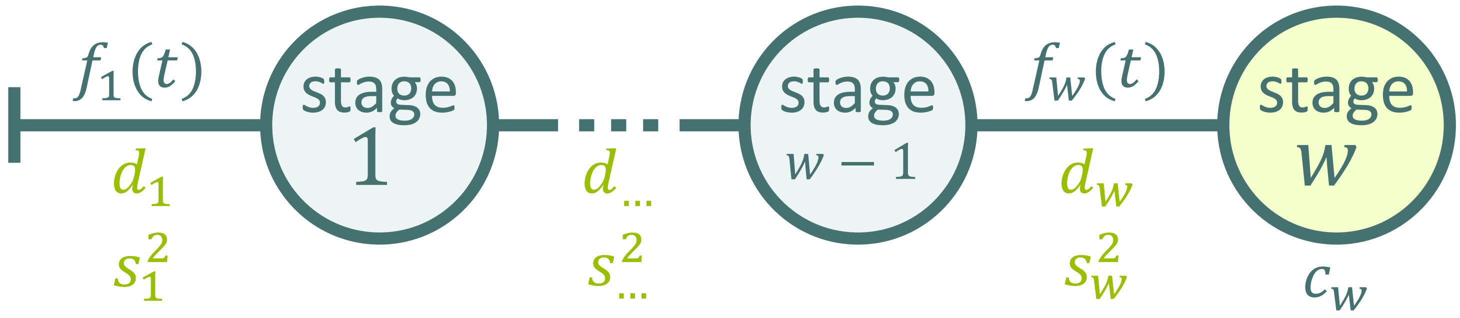

- The mean and variance of the distribution of time until cash flow c_w is incurred is:
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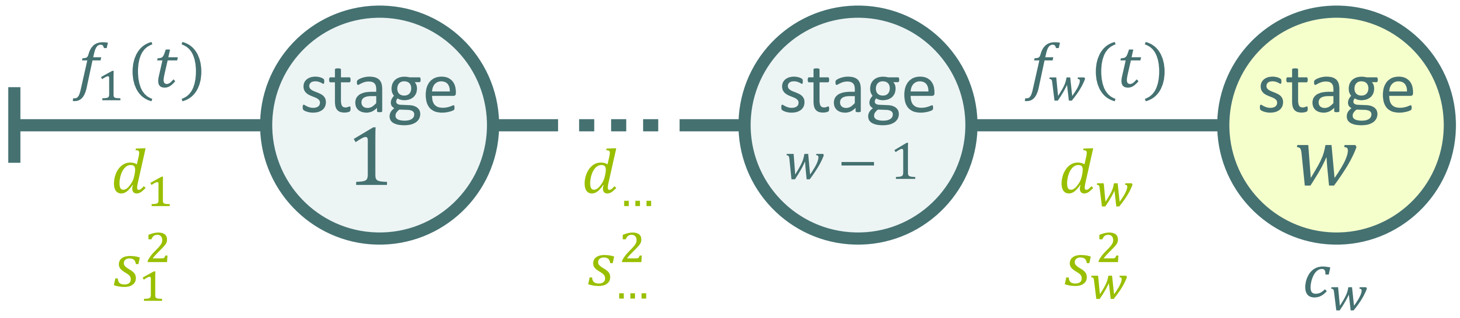

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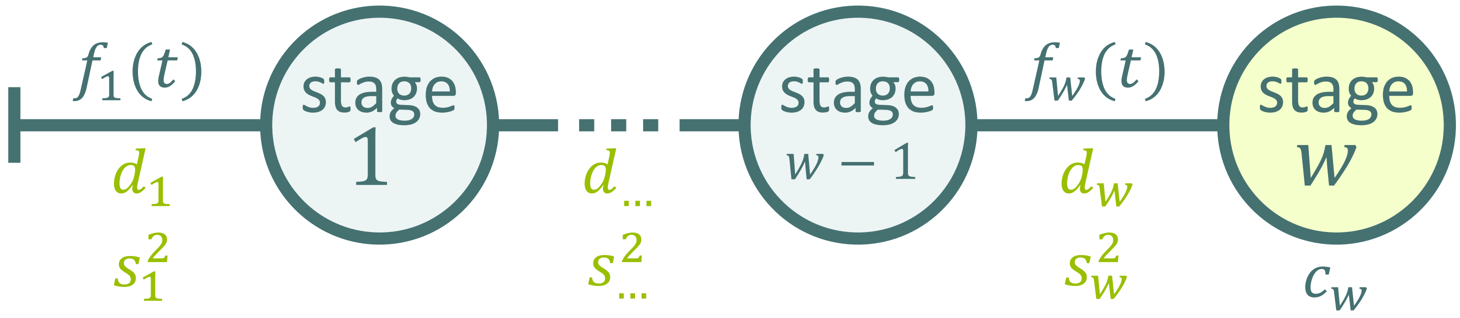

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- If $f_{1,w}(t)$ is normally distributed, the NPV of cash flow c_w is lognormally distributed!

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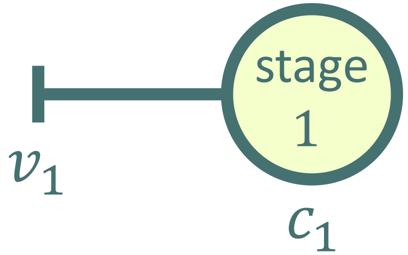

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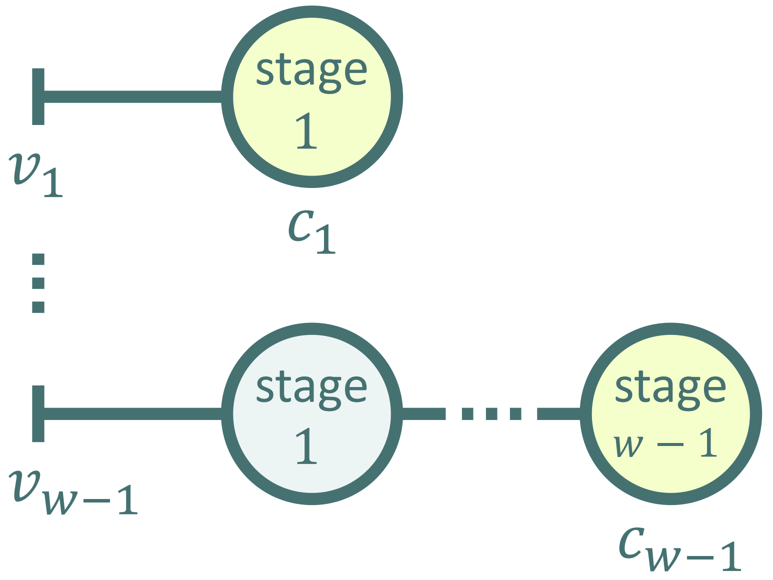

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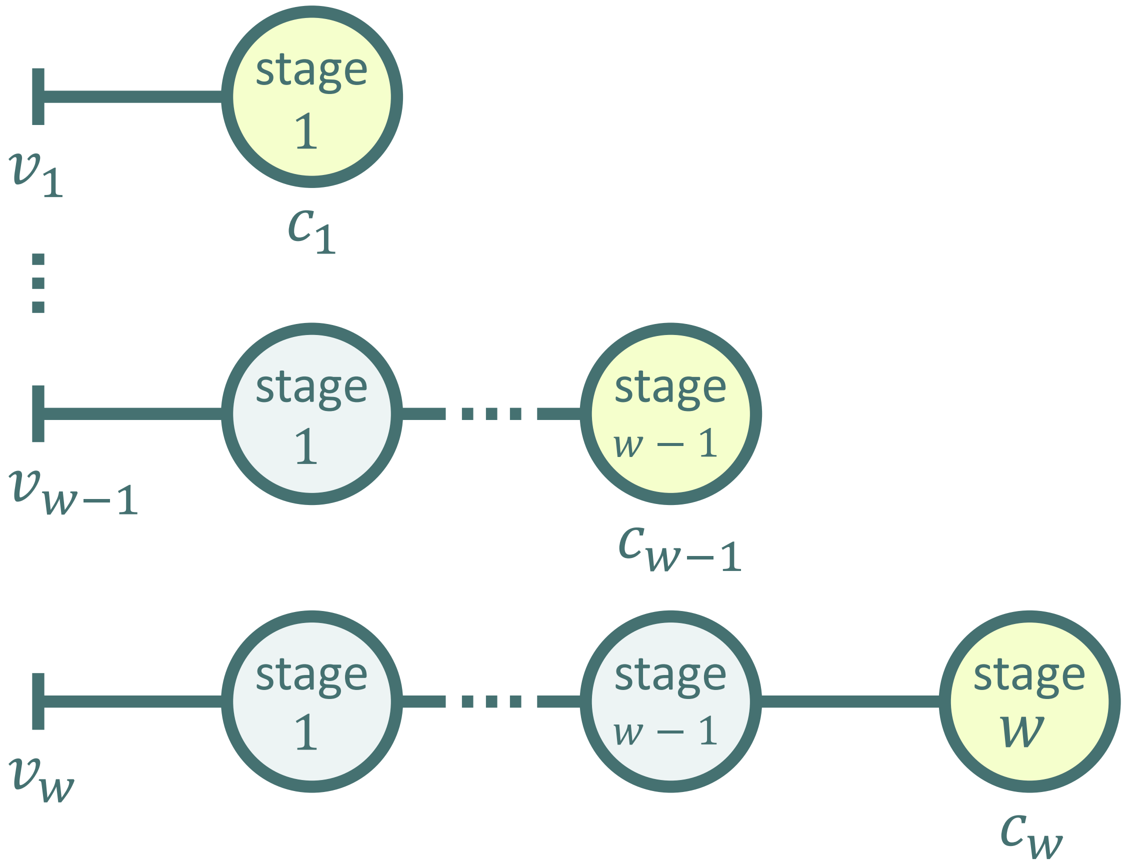

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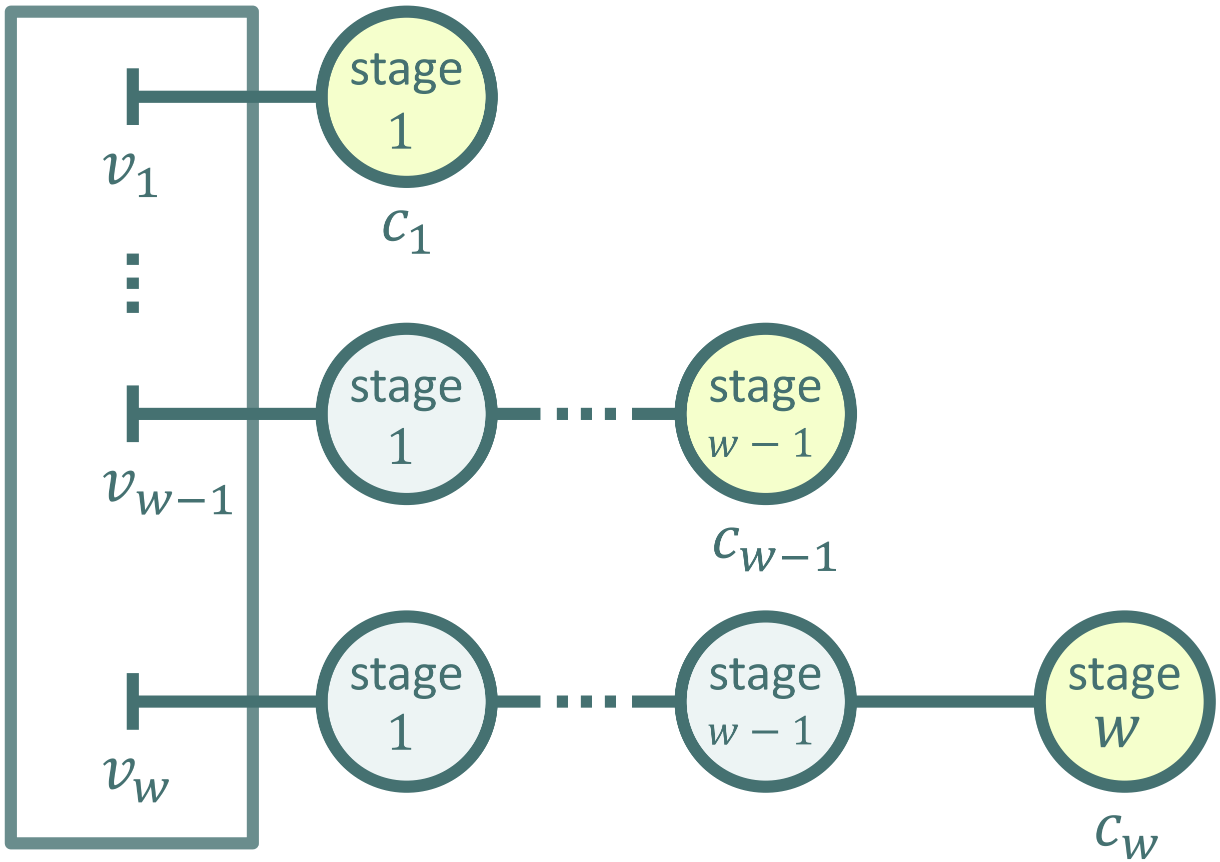

$v = v_1 + \ldots + v_{w-1} + v_w$

NPV of a serial project

We can obtain the moments of the NPV of the serial project using exact, closed-form formula's:

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Mean μ
$\mu_w = c_w a_1$

Covariance matrix $\boldsymbol{\Sigma}_c$
$\boldsymbol{\Sigma}_c(w,w) = \sigma_w^2 = c_w^2(a_2 - a^2)$
$\boldsymbol{\Sigma}_c(w,x) = c_w c_x b_1 \left(a_2 - a^2\right) = c_w^{-1} c_x b_1 \boldsymbol{\Sigma}_c(w,w)$

Central coskewness matrix $\boldsymbol{\Gamma}_c$
$\boldsymbol{\Gamma}_c(w,w,w) = \gamma_w \sigma_w^3 = c_w^3 \left(a_3 - 3a_2 a_1 + 2a^3\right)$
$\boldsymbol{\Gamma}_c(w,w,x) = c_w^{-1} c_x b_1 \boldsymbol{\Gamma}_c(w,w,w)$
$\boldsymbol{\Gamma}_c(w,x,x) = c_w c_x^2 \left(a_3 b_2 - a_2 a_1 \left(2b^2 + b_2\right) + 2a^3 b^2\right)$
$\boldsymbol{\Gamma}_c(w,x,y) = c_x^{-1} c_y h_1 \boldsymbol{\Gamma}_c(w,x,x)$

Central cokurtosis matrix $\boldsymbol{\Theta}_c$
$\boldsymbol{\Theta}_c(w,w,w,w) = \theta_w \sigma_w^4 = c_w^4 \left(a_4 - 4a_3 a_1 + 6a_2 a^2 - 3a^4\right)$
$\boldsymbol{\Theta}_c(w,w,w,x) = c_w^{-1} c_x b_1 \boldsymbol{\Theta}_c(w,w,w,w)$
$\boldsymbol{\Theta}_c(w,w,x,x) = c_w^2 c_x^2 \left(a_4 b_2 - 2a_3 a_1 \left(b_2 + b^2\right) + a_2 a^2 \left(b_2 + 5b^2\right) - 3a^4 b^2\right)$
$\boldsymbol{\Theta}_c(w,x,x,x) = c_w c_x^3 \left(a_4 b_3 - a_3 a_1 \left(b_3 + 3b_2 b_1\right) + 3a_2 a^2 \left(b_2 b_1 + b^3\right) - 3a^4 b^3\right)$
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$\boldsymbol{\Theta}_c(w,x,y,y) = c_w c_x c_y^2 \left(\left(a_4 - a_3 a_1\right) b_3 h_2 - \left(h_2 + 2h^2\right) \left(\left(a_3 a_1 - a_2 a^2\right) b_2 b_1\right) + \left(a_2 a^2 - a^4\right) 3b^3 h^2\right)$
$\boldsymbol{\Theta}_c(w,x,y,z) = c_y^{-1} c_z o_1(r) \boldsymbol{\Theta}_c(w,x,y,y)$

$a_i = \phi_{1,w-1}(ir)$	$b_i = \phi_{w,x-1}(ir)$	$h_i = \phi_{x,y-1}(ir)$	$o_i = \phi_{y,z-1}(ir)$
$a^i = \phi^i_{1,w-1}(r)$	$b^i = \phi^i_{w,x-1}(r)$	$h^i = \phi^i_{x,y-1}(r)$	

NPV of a serial project

We develop a three-parameter lognormal distribution that can be used to match the mean, variance, and skewness of the true NPV distribution

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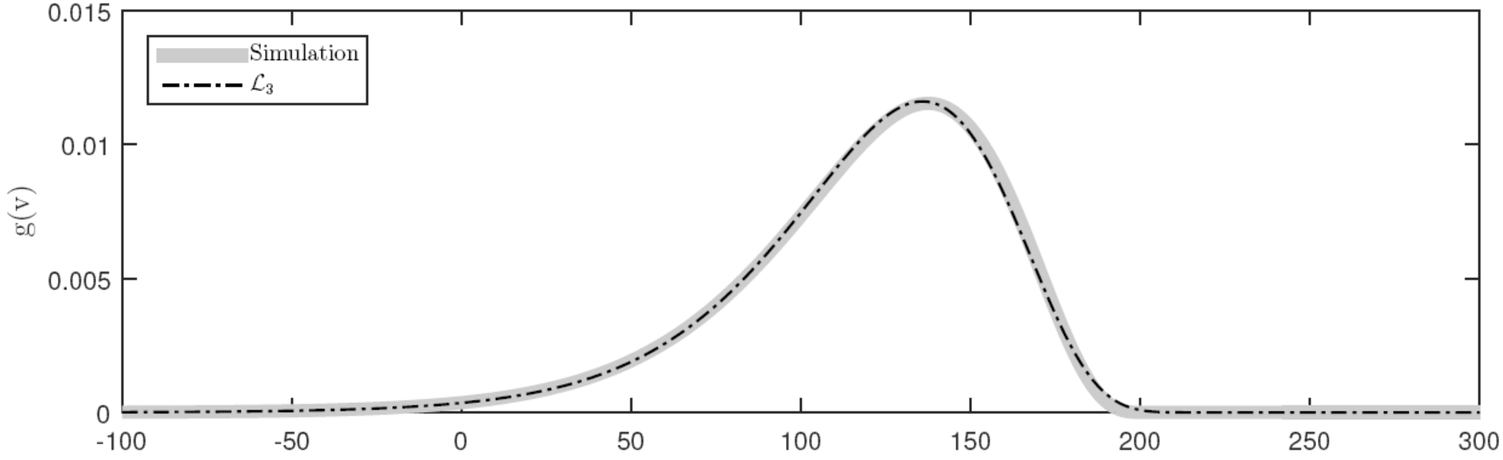
The example below illustrates the accuracy of the three-parameter lognormal distribution ($\mathcal{L}_3$):



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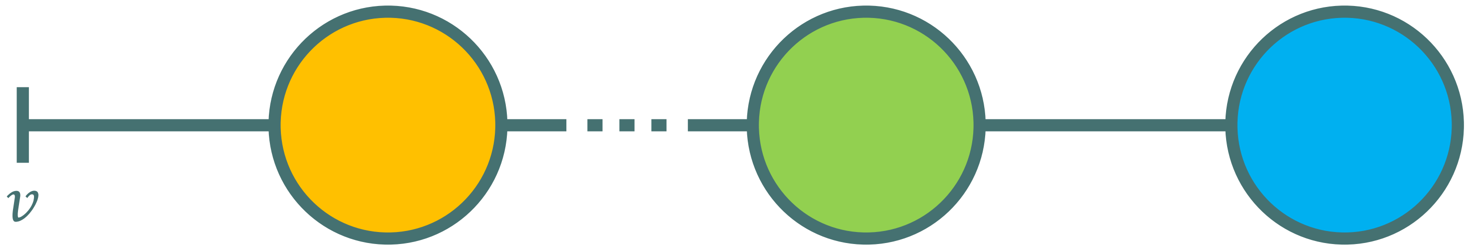
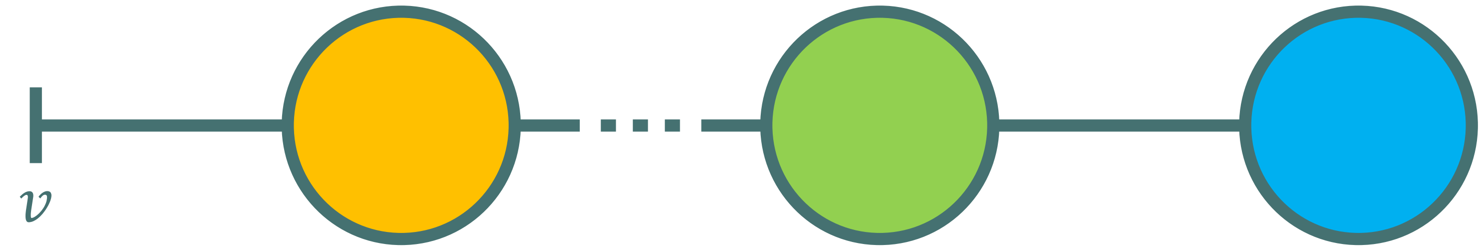
Optimal sequence of stages



- Moments of known sequence can be obtained using exact closed-form formulas

Optimal sequence of stages



- Moments of known sequence can be obtained using exact closed-form formulas
- How to obtain the optimal sequence of a set of stages that are potentially precedence related?

Optimal sequence of stages

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Optimal sequence of stages

- The problem to find the optimal sequence of stages is equivalent to the Least Cost Fault Detection Problem (LCFDP)
- The LCFDP minimizes the cost of the sequential diagnosis of a number of system components
- In the absence of precedence relations, the optimal sequence can be found in polynomial time
- Efficient algorithms are available for the general case

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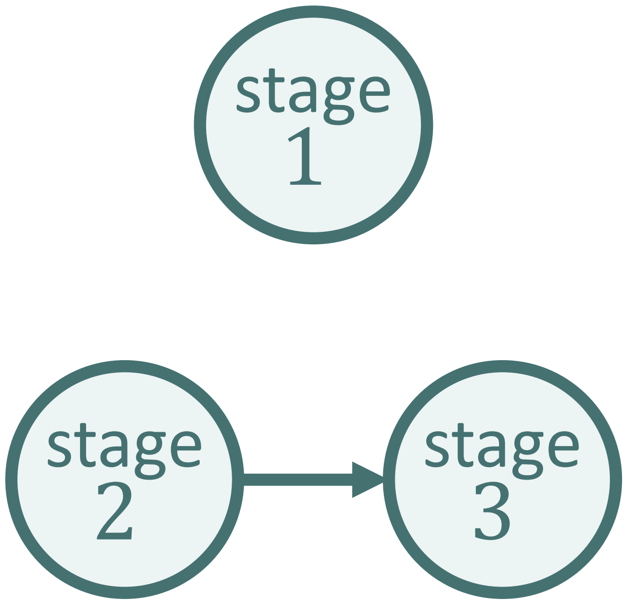
NPV of a general project

NPV of a general project
Scheduling policies



NPV of a general project
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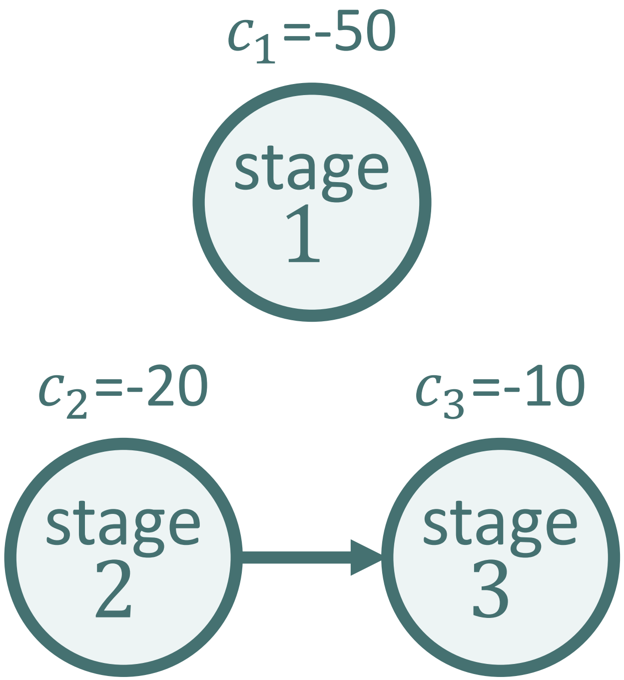

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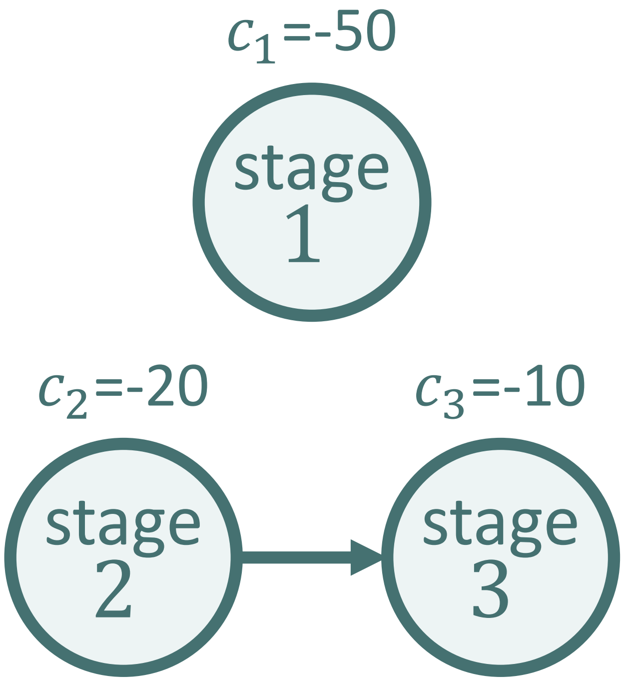

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NPV of a general project
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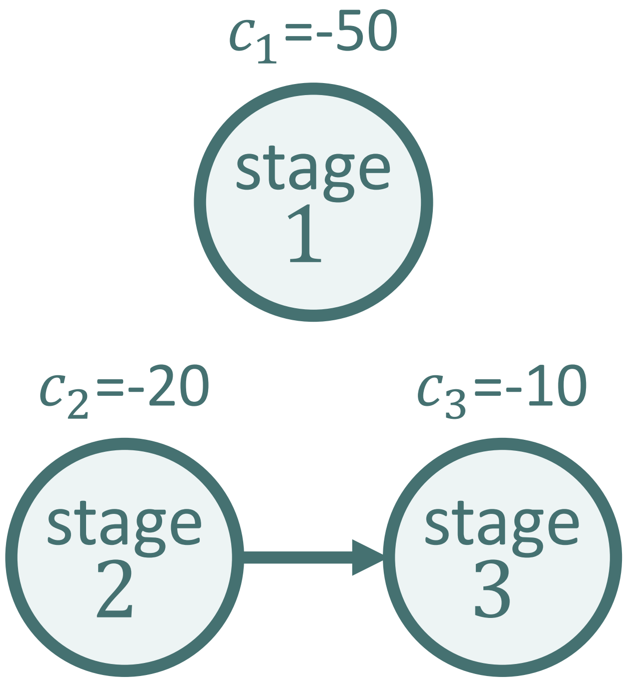

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NPV of a general project
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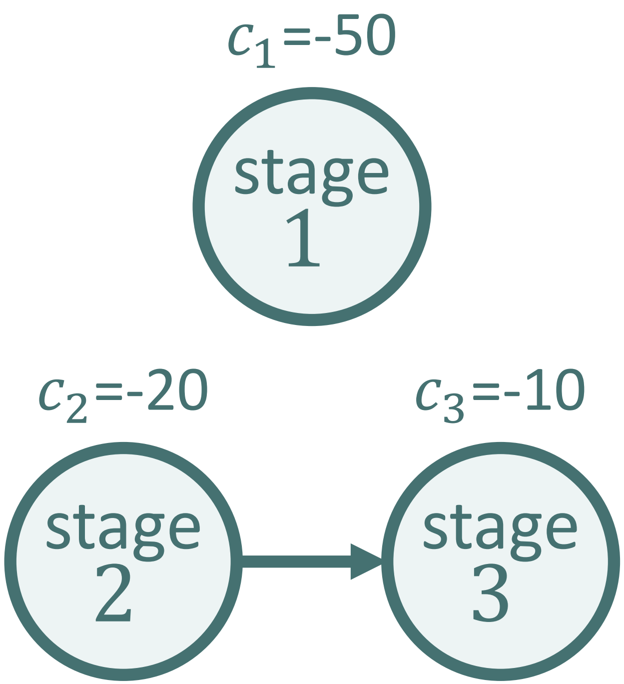

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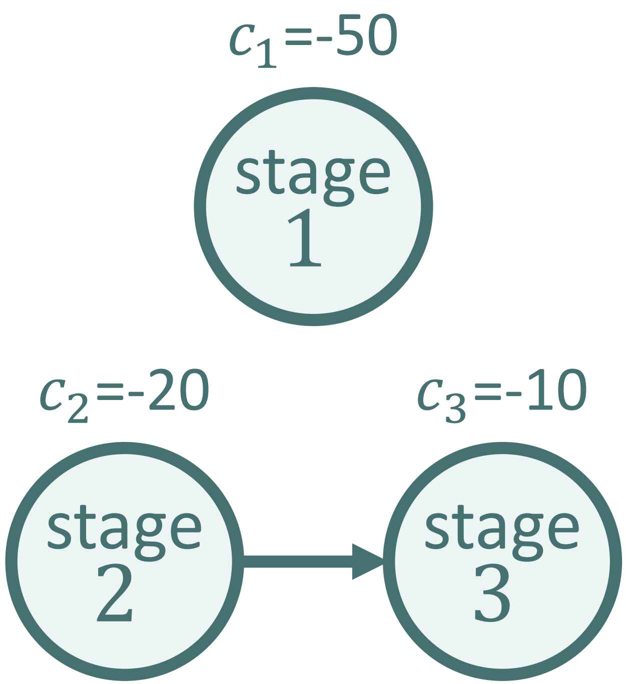

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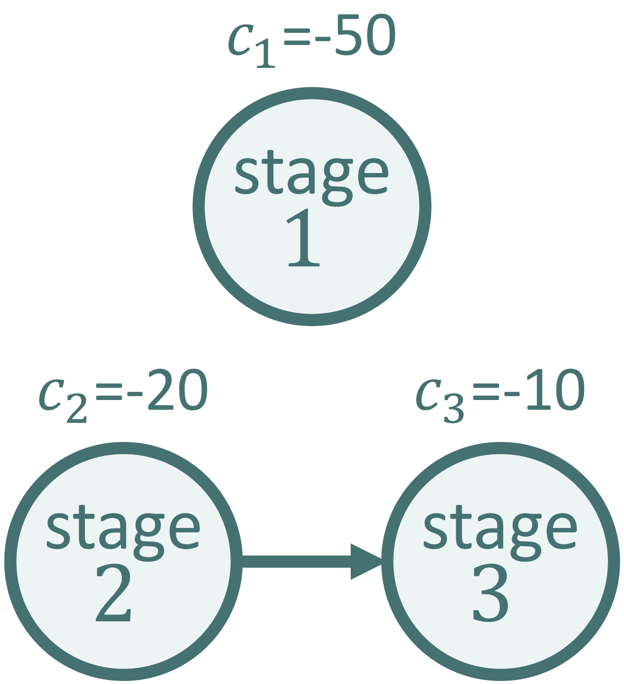
  - 1-3-2
  - 2-1-3
  - 2-3-1
  - 3-1-2
  - 3-2-1

NPV of a general project
Scheduling policies



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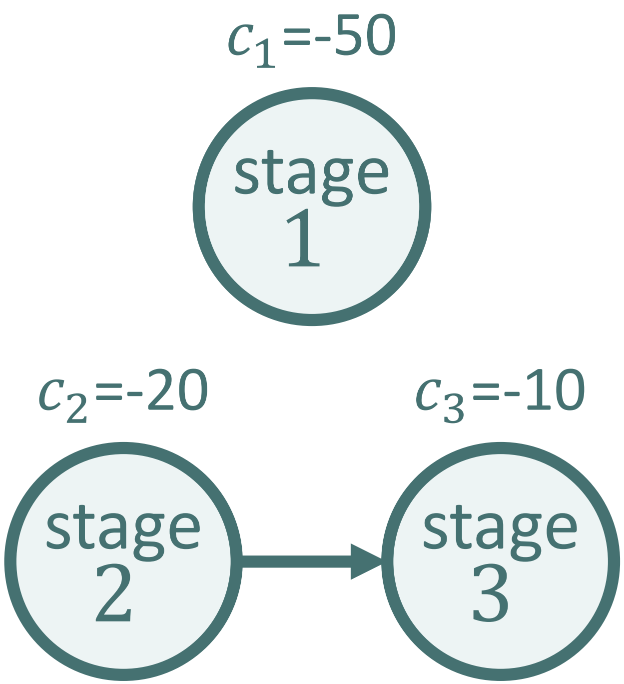
  - 1-3-2
  - 2-1-3
  - 2-3-1
  - 3-1-2
  - 3-2-1
- Early-Start (ES) policy: Start 1 & 2. Start 3 upon completion of 2.

NPV of a general project
Scheduling policies



$f_1(t) \sim Exp(1)$

$f_{2,3}(t) \sim Exp(0.5)$

$p = 200 \quad r = 0.1$

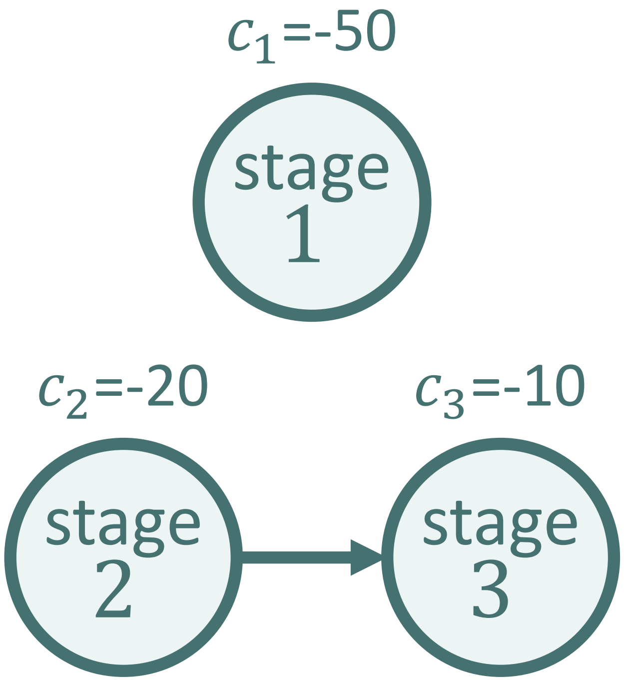
- Serial policies:
  - 1-2-3
  - 1-3-2
  - 2-1-3
  - 2-3-1
  - 3-1-2
  - 3-2-1
- Early-Start (ES) policy: Start 1 & 2. Start 3 upon completion of 2.

…

- Optimal policy: Start 2. Start 1 & 3 upon completion of 2.

NPV of a general project

Early-Start policy



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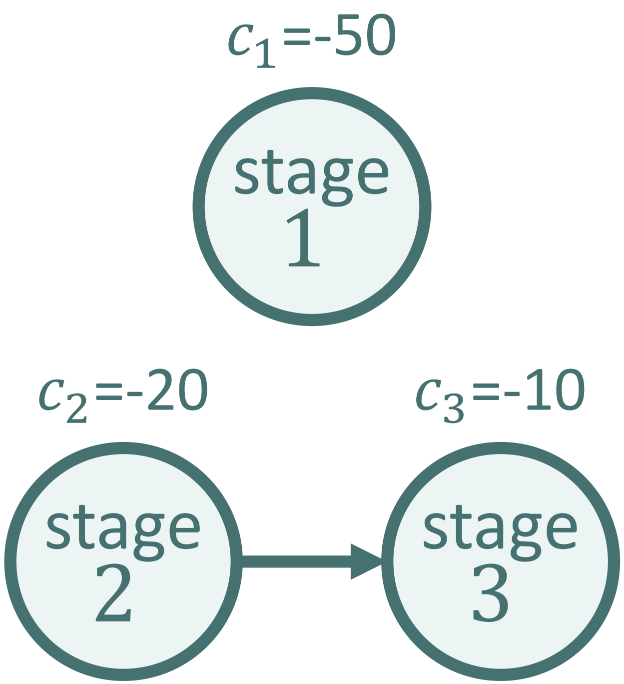

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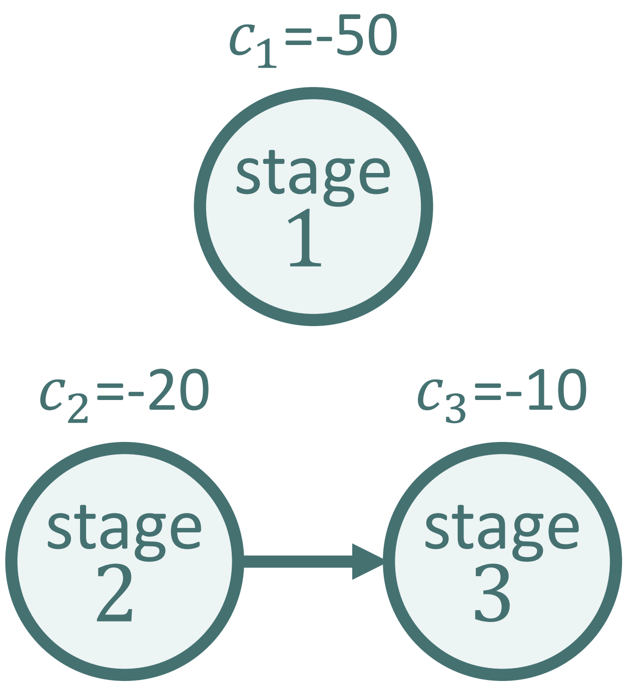
  - After stage 1?
  - After stage 2&3?

NPV of a general project

Early-Start policy



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- What discount factor do we use?
  - $\phi_1(r)$
  - $\phi_{2,3}(r)$

NPV of a general project

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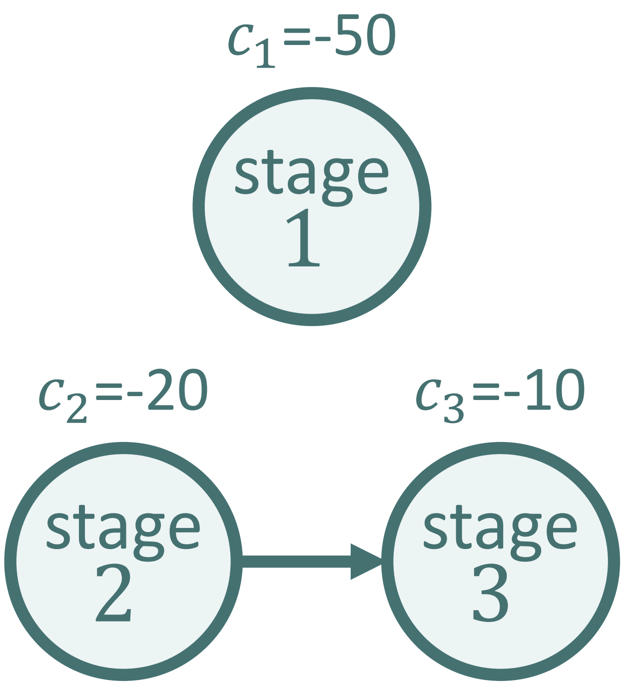

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NPV of a general project

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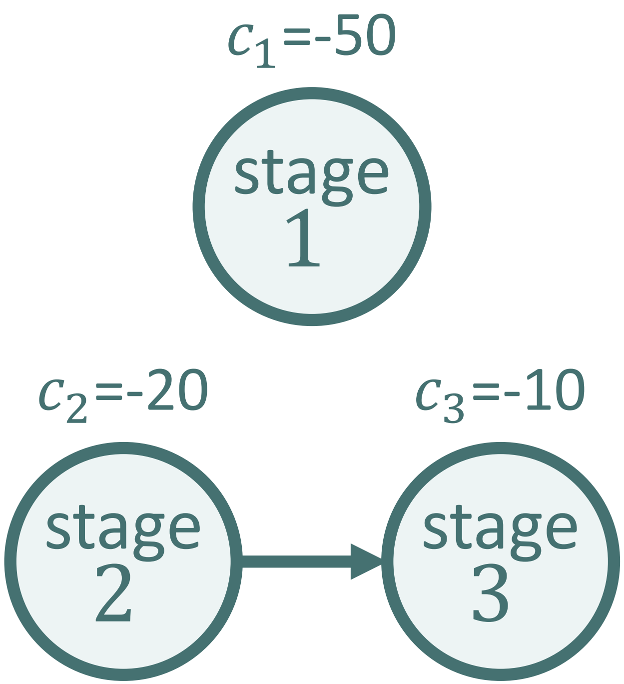

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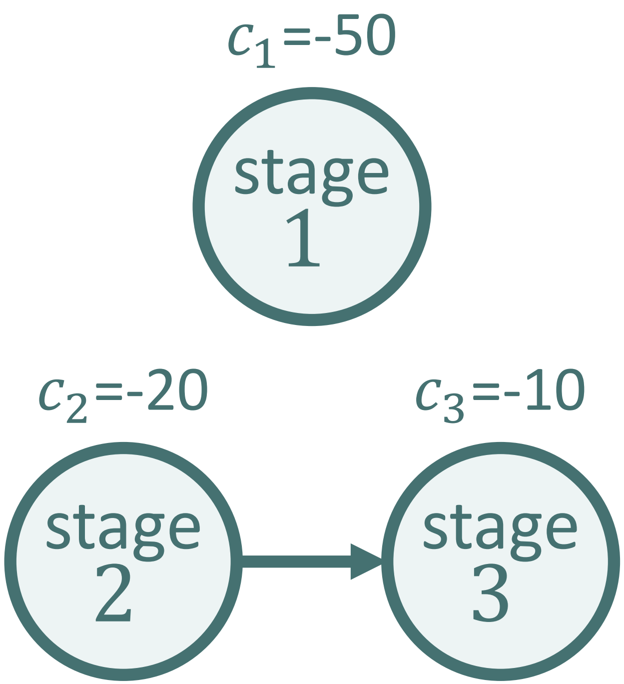
$\Rightarrow$Approximations are required!

NPV of a general project

Optimal policy



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NPV of a general project

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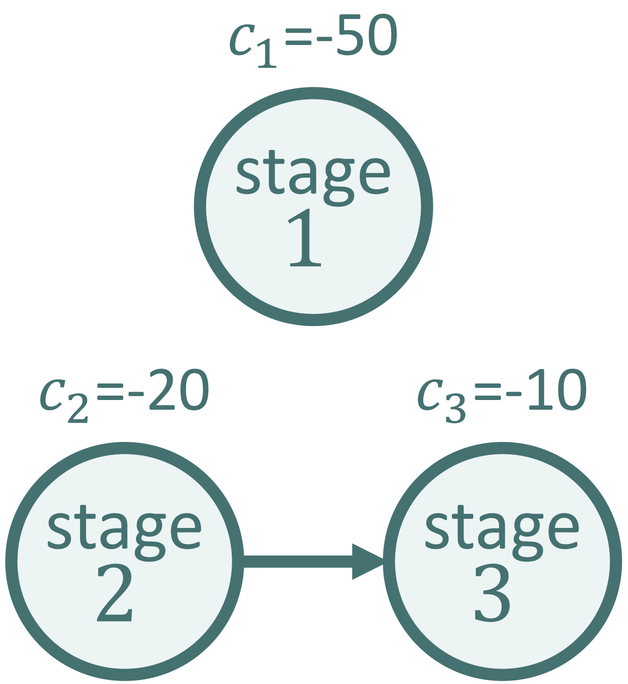

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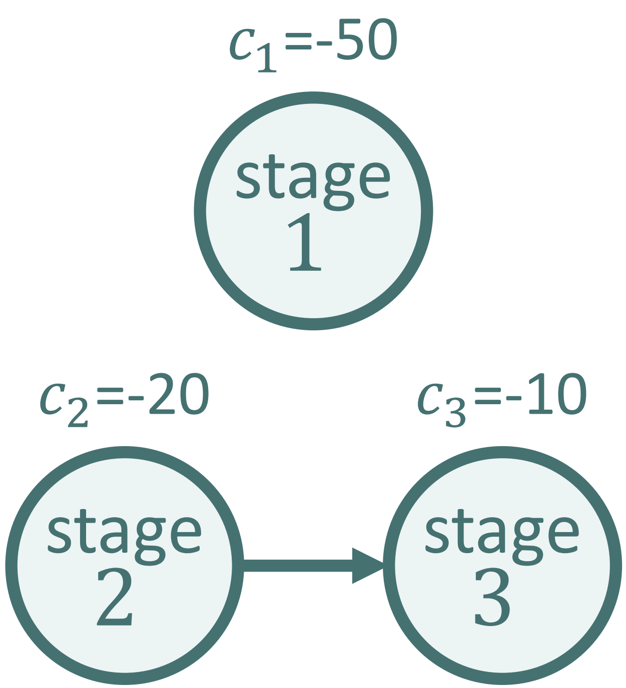
NPV of a general project

Optimal policy



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NPV of a general project

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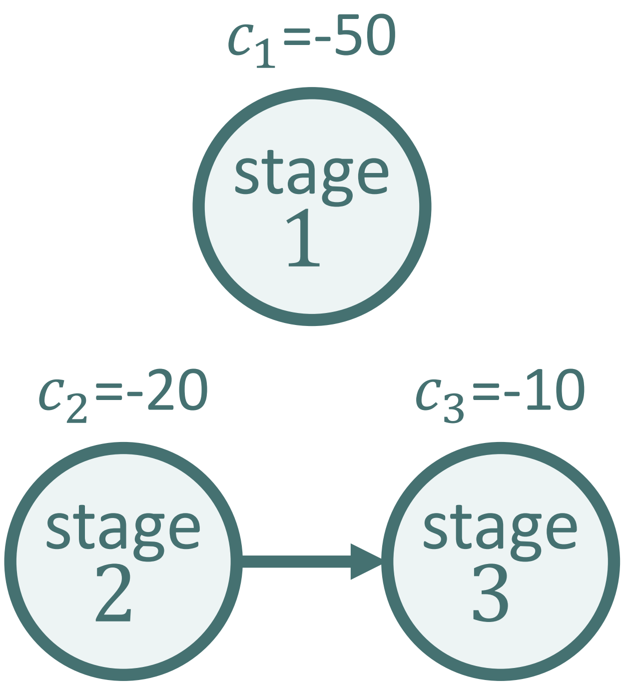

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NPV of a general project

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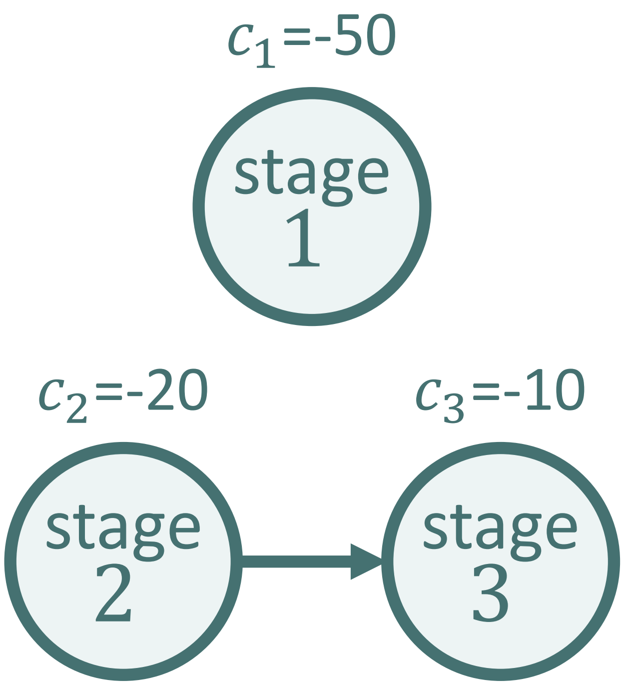

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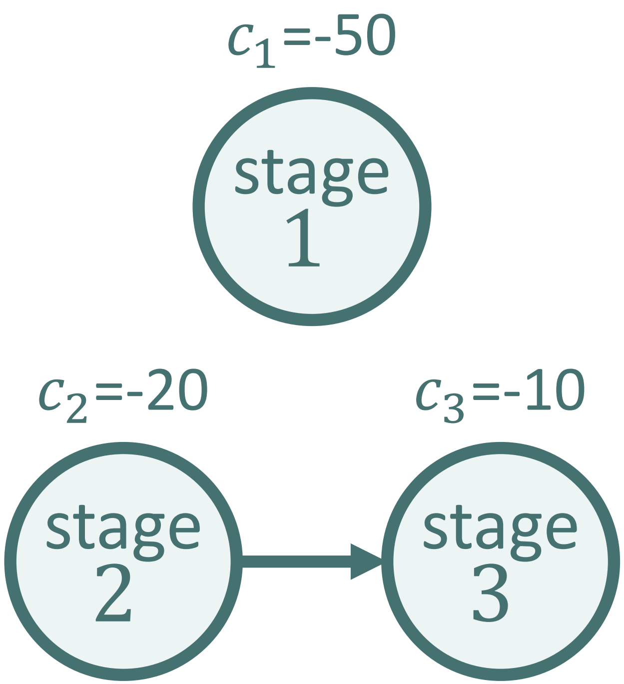
NPV of a general project

Optimal policy



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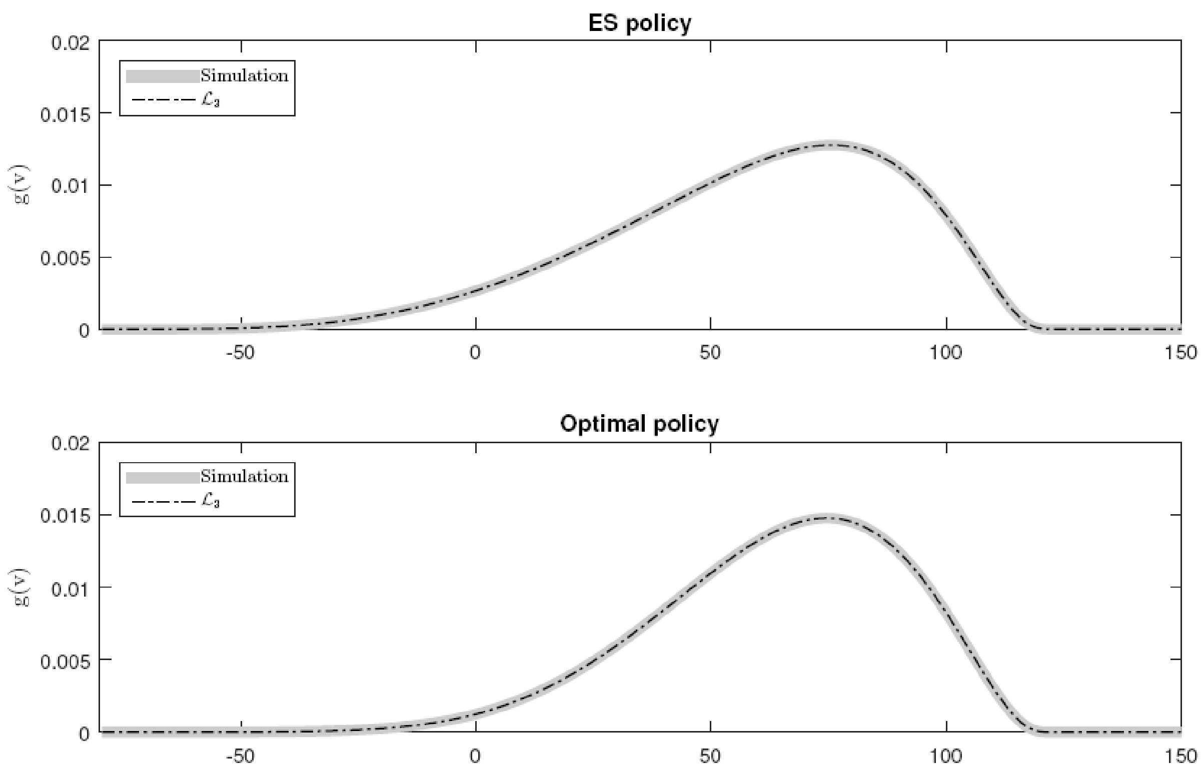
$\Rightarrow$ If this is not possible, approximations are required!

NPV of a general project

The example below illustrates the accuracy of the three-parameter lognormal distribution ($\mathcal{L}_3$) for the ES and the optimal policy:



Agenda

- Introduction
- Serial projects:
  - Single cash flow after a single stage
  - Single cash flow after multiple stages
  - NPV of a serial project
  - Optimal sequence of stages
- General projects
- Conclusions

Conclusion

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- The optimal sequence of stages can be found efficiently
- The eNPV of a general project can be obtained using exact, closed-form expressions

Conclusion

- We obtain exact, closed-form expressions for the moments of the NPV of serial projects
- The distribution of the NPV of a serial project can be approximated accurately using a three-parameter lognormal distribution
- The optimal sequence of stages can be found efficiently
- The eNPV of a general project can be obtained using exact, closed-form expressions
- Higher moments & the distribution of the NPV of a general project can be approximated

TIME FOR QUESTIONS?


VIVE LA
MANNSCHAFT
Benz