A new approach for quantitative risk analysis

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Abstract - **Project risk management aims to provide insight into the risk profile of a project as to facilitate decision makers to mitigate the impact of risks on project objectives such as budget and time. A popular approach to determine where to focus mitigation efforts, is the use of so-called ranking indices (e.g., the criticality index, the significance index etc.). Ranking indices allow the ranking of project activities (or risks) based on the impact they have on project objectives. A distinction needs to be made between activity-based ranking indices (those that rank activities) and risk-driven ranking indices (those that rank risks). Because different ranking indices result in different rankings of activities and risks, one might wonder which ranking index is best. In this article, we provide an answer to this question. Our contribution is threefold: (1) we set up a large computational experiment to assess the efficiency of ranking indices in the mitigation of risks, (2) we develop two new ranking indices that outperform existing ranking indices and (3) we show that a risk-driven approach is more effective than an activity-based approach.**

Keywords - **project risk management, risk mitigation, ranking index**

1 Introduction

It is well known that projects worldwide are still struggling to meet their objectives (The Standish Group 2009). During project execution, unforeseen events arise that disrupt plans and budgets and that result in substantial overruns. Risk management is widely recognized as a compulsory discipline to deal with this kind of project uncertainty.

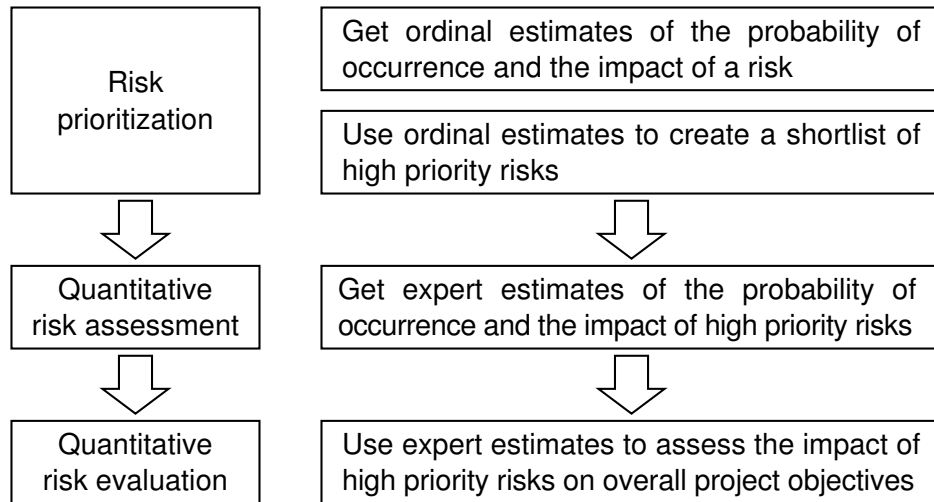| Risk prioritization | Get ordinal estimates of the probability of occurrence and the impact of a risk |
	Use ordinal estimates to create a shortlist of high priority risks
Quantitative risk assessment	Get expert estimates of the probability of occurrence and the impact of high priority risks
Quantitative risk evaluation	Use expert estimates to assess the impact of high priority risks on overall project objectives

Figure 1: Overview of the risk analysis process

The Project Management Institute (2008) defines risk management as the process that deals with the planning, identification, analyzing, responding, monitoring and controlling of project risks. In this article, we focus on the risk analysis process and its effect on the risk response process. The risk analysis process can be divided into a number of subprocesses: risk prioritization, quantitative risk assessment and quantitative risk evaluation. Risk prioritization is a qualitative procedure that allows to prioritize the risks that were identified in an earlier stage of the risk management process. It requires ordinal estimates of both the probability of occurrence and the impact of a risk. These ordinal estimates are then used to create a shortlist of high priority risks (analogous to the Pareto principle). Further risk analysis efforts should focus on these high priority risks. Quantitative risk assessment is the procedure in which experts provide detailed estimates of the probability of occurrence and the impact of high priority risks. These estimates are used in the quantitative risk evaluation procedure to analyze the impact of the shortlisted risks on overall project objectives. Figure 1 provides a short overview of the dynamics of the risk analysis process.

Good risk management requires a risk analysis process that is scientifically sound and that is supported by quantitative techniques (Hubbard 2008). A wide body of knowledge on quantitative techniques has been accumulated

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over the last two decades. Monte Carlo simulation is the predominant quantitative risk evaluation technique in both practice and in literature. Advocates of alternative techniques such as neural networks, fuzzy logic and decision tree analysis have debatable arguments in favor of these techniques, but have so far failed to persuade most project schedulers of their practical use (refer to Sadeghi et al. (2009) and Georgieva et al. (2009) among others for an evaluation of risk analysis techniques).

The goal of the risk analysis process is to generate insight into the risk profile of a project and to use these insights to drive the risk response process (The Project Management Institute 2008). The insights generated include: the probability of achieving a specific project outcome, the distribution function of the project completion time, etc. The risk response process will use these insights to define practical risk responses that allow project managers to mitigate risks (i.e., to reduce the impact of risks on project objectives). A popular approach to determine where to focus mitigation efforts is the use of so-called ranking indices (e.g., the criticality index, the significance index, etc.). Ranking indices allow the ranking of project activities (or risks) based on their impact on project objectives. A distinction needs to be made between activity-based ranking indices (those that rank activities) and risk-driven ranking indices (those that rank risks). Note that the impact of an activity (or risk) on a project objective may differ depending on the ranking index used, resulting in the question: which ranking index is best? It is exactly this question that we will address in this article.

The contribution of this article is threefold: (1) we set up a large computational experiment to assess the mitigation efficiency of ranking indices, (2) we develop two new ranking indices that outperform existing ranking indices and (3) we show that a risk-driven approach is more effective than an activity-based approach. We assume risks to impact the duration of activities and hence use the project completion time to evaluate the performance of ranking indices (i.e., we assess the potential of ranking indices to mitigate risks that delay the completion time of a project). In order to approximate the distribution of the project completion time, we adopt Monte Carlo simulation.

The remainder of this article is organized as follows: in section 2 we review the basic principles of stochastic project scheduling. Section 3 introduces the risk-driven approach and compares it to the activity-based approach. Section 4 presents the ranking indices. The computational experiment as well as the performance of the ranking indices are discussed in section 5. Section 6

presents a number of additional experiments and section 7 concludes. A list of notation is provided in the appendix.

2 Stochastic project scheduling

The Critical Path Method (CPM) is developed in the 50's by DuPont Corporation and provides the foundations of modern project scheduling (Kelley 1963). It represents a project as an activity network which is a graph $G = (N, A)$ that consists of a set of nodes $N = \{1, 2, \ldots, n\}$ and a set of arcs $A = \{(i, j)|i, j \in N\}$. The nodes represent project activities whereas the arcs that connect the nodes represent precedence relationships. Activities 1 and n are referred to as the dummy-start and the dummy-end activity and represent the start and the completion of the project respectively. Each activity j has a deterministic activity duration d_j and can only start when its predecessors have finished. CPM adopts an early-start schedule in which activities are scheduled to start as soon as possible. The early-start schedule may be represented by a vector of earliest start times $\mathfrak{s} = \{s_1, s_2, \ldots, s_n\}$. The earliest start time of an activity j is defined as follows:

$$s_j = \max\left\{f_i|(i, j) \in A\right\}, \tag{1}$$

where f_i is the earliest finish time of an activity i and equals:

$$f_i = s_i + d_i. \tag{2}$$

By convention, the project starts at time instance 0 (i.e., $s_1 = 0$). According to CPM, the project completion time c is given by:

$$c = f_n. \tag{3}$$

The longest path of the scheduled activities is called the critical path and the activities on this path are critical activities.

Since the establishment of CPM, many extensions of the basic model have been introduced: generalized precedence relationships, resource-constrained project scheduling, multi-mode scheduling, critical chain buffer management, etc. We refer to Demeulemeester and Herroelen (2002) for an extensive overview of the field. In this article, we are particularly interested in what is called stochastic project scheduling or stochastic CPM. For an overview of the recent developments in stochastic project scheduling, refer to Elmaghraby

(2000 and 2005), Bendavid and Golany (2009), Abdelkader (2010) and the references therein. Stochastic CPM acknowledges that activity durations are not deterministic. We model the duration of an activity j as a positive random variable D_j. Because the duration of an activity is a random variable, the earliest start and finish times of an activity are random variables as well. Let S_j and F_j denote the random variable of the earliest start and finish times of an activity j respectively. The project completion time is a random variable C which is a function of D_j. Calculating the distribution function of C is proven to be $\#P$-complete (Hagstrom 1988) and thus requires approximative methods such as Monte Carlo simulation (Van Slyke 1963). Monte Carlo simulation is used to virtually execute a project a large number of times, providing insight and allowing the project manager to enhance the actual execution of the project.

We rely on Monte Carlo simulation to obtain random variates of D_j. Let $\mathbf{d}_j = \{d_{j,1}, d_{j,2}, \ldots, d_{j,q}\}$ denote the vector of q random variates of D_j (where q represents the number of simulation iterations). We refer to $\mathbf{d}_j$ as the vector of realized durations of D_j. In addition, define $\mathbf{s}_j$ the vector of realized earliest start times of an activity j:

$$\mathbf{s}_j = \max\left\{\mathbf{f}_i | (i, j) \in A\right\}, \tag{4}$$

where $\mathbf{f}_i$ is the vector of realized earliest finish times of an activity i and equals:

$$\mathbf{f}_i = \mathbf{s}_i + \mathbf{d}_i. \tag{5}$$

The vector of realized project completion times $\mathbf{c}$ is defined as follows:

$$\mathbf{c} = \mathbf{f}_n. \tag{6}$$

It is clear that $\mathbf{s}_j$, $\mathbf{f}_j$ and $\mathbf{c}$ are vectors of random variates of random variables S_j, F_j and C respectively.

3   Towards a risk-driven approach

One of the main challenges in project risk management is to estimate and to model the uncertainty of activity durations. Often, it is assumed that the duration of an activity follows a distribution that captures all uncertainty that originates from the occurrence of risks (popular distributions include: the triangular distribution, the beta distribution and the normal distribution).

As such, risk assessment boils down to providing estimates of activity duration distribution parameters. We refer to this approach as the activity-based approach.

In this article, we argue that the activity-based approach is inherently flawed. As Hulett (2009) points out, there is no clear link between the impact of identified risks on the duration of an activity and the distribution of the activity duration itself (i.e., the activity-based approach is unable to identify the root causes of the uncertainty in the duration of an activity). In addition, our experience learns that practitioners have a hard time assessing uncertainty by estimating the parameters of an activity duration distribution.

To resolve the problems of the activity-based approach, we devise a risk-driven approach in which the impact of each risk is assessed individually and is mapped to the duration of an activity afterwards. Our approach is based on previous work by Schatteman et al. (2008) and Van de Vonder (2006) and is similar to the risk-driver approach of Hulett (2009). Contrary to the activity-based approach, we focus on risks as primary sources of uncertainty. In what follows, we adopt an integrated approach that relies on Monte Carlo simulation to evaluate the impact of risks on activity durations and on the project completion time. Figure 2 presents a visual overview from which it is clear that a risk-driven approach assesses the impact of root risks on the uncertainty of the activities and on the project completion time. An activity-based approach, on the other hand, assesses only the uncertainty of the activities without observing the root risks that cause this uncertainty.

To further support the risk-driven approach, we provide the following example. Consider an activity whose duration is impacted by two risks. The first risk has a small impact yet a large probability of occurrence whereas the second risk has a large impact but a small probability of occurrence. The probability distribution of the duration of the activity is presented in figure 3. From the figure, it is clear that fitting a distribution would result in significant errors (the best fit of the triangular distribution is indicated by the dotted line). In addition, it would be very hard for practitioners to estimate the parameters of the fitted distribution. Assessing the probability of occurrence and the impact of both risks on the other hand, would be a manageable task and would result in the correct distribution of the duration of the activity.

In order to formally define risks and their impacts, let $R = \{1, 2, \ldots, r\}$ denote the set of risks and let $\mathbf{M} = \{M_{j,e} | j \in N \wedge e \in R\}$ denote the set of risk impacts, where $M_{j,e}$ is the random variable of the risk impact of a risk
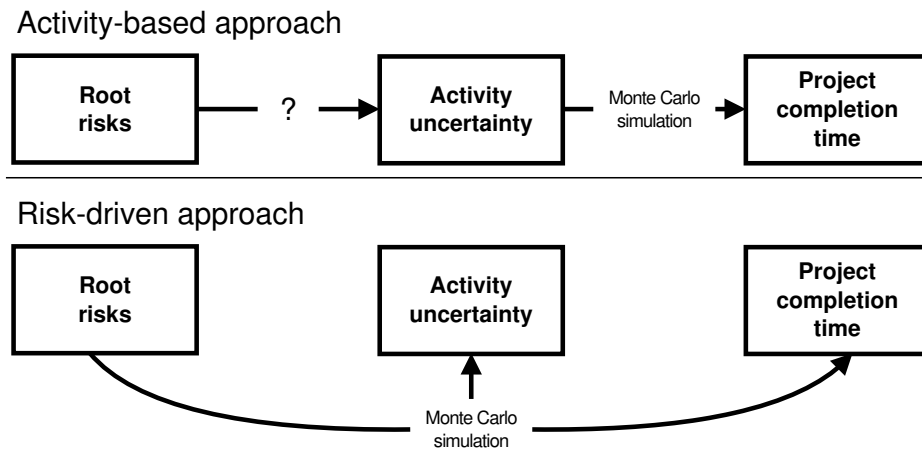
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Activity-based approach

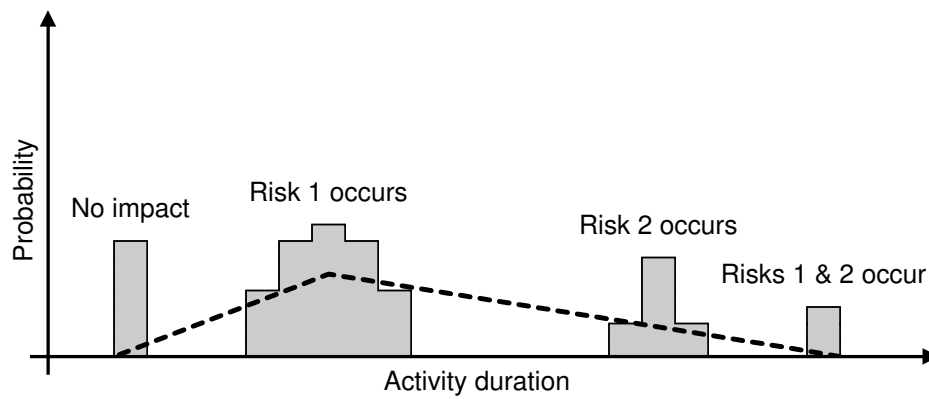Figure 2: Activity-based versus risk-driven approach

Figure 3: Example distribution of the duration of an activity

e on the duration of an activity j. Let $\mathbf{m}_{j,e} = \{m_{j,e,1}, m_{j,e,2}, \ldots, m_{j,e,q}\}$ represent the vector of random variates of $M_{j,e}$ and define $\mathbf{d}_j^{(E)} = \left\{ d_{j,1}^{(E)}, d_{j,2}^{(E)}, \ldots, d_{j,q}^{(E)} \right\}$, the vector of random variates of the duration of an activity j when subject to a set of risks $E \subseteq R$. The entries of $\mathbf{d}_j^{(E)}$ are computed as follows:

$$d_{j,p}^{(E)} = d_j + \sum_{e \in E} m_{j,e,p} \qquad \forall p \in \{1, 2, \ldots, q\}, \tag{7}$$

where d_j is the deterministic duration of an activity j (i.e., d_j represents the duration of an activity j when it is not impacted by risks). From $\mathbf{d}_j^{(E)}$, we obtain $\mathbf{s}_j^{(E)} = \left\{ s_{j,1}^{(E)}, s_{j,2}^{(E)}, \ldots, s_{j,q}^{(E)} \right\}$, $\mathbf{f}_j^{(E)} = \left\{ f_{j,1}^{(E)}, f_{j,2}^{(E)}, \ldots, f_{j,q}^{(E)} \right\}$ and $\mathbf{c}^{(E)} = \left\{ c_1^{(E)}, c_2^{(E)}, \ldots, c_q^{(E)} \right\}$ by generalizing equations 4, 5 and 6:

$$\mathbf{s}_j^{(E)} = \max\left\{ \mathbf{f}_i^{(E)} | (i, j) \in A \right\}, \tag{8}$$

$$\mathbf{f}_j^{(E)} = \mathbf{s}_j^{(E)} + \mathbf{d}_j^{(E)}, \tag{9}$$

$$\mathbf{c}^{(E)} = \mathbf{f}_n^{(E)}. \tag{10}$$

The expected project delay over q simulation iterations is defined as follows:

$$\Delta^{(E)} = \frac{1}{q} \sum_{p=1}^{q} c_p^{(E)} - c, \tag{11}$$

where c is the deterministic project completion time and is obtained using equation 3.

4 Effective risk mitigation

Most commercial risk analysis software packages provide the functionality to generate insight into the source of project overruns. The activities (or the risks) that contribute most to the project delay are identified using ranking indices. Let $(\cdot)_j^{(E)}$ and $(\cdot)_e^{(E)}$ denote the ranking values of a ranking index $(\cdot)$ for an activity j and a risk e when activity durations are subject to a set of risks E. The larger the ranking value, the larger the contribution of the activity (or the risk) to the project delay. The ranking of activities (or risks) is typically visualized using a ranked bar chart (see figure 4 for an example of a ranked bar chart).
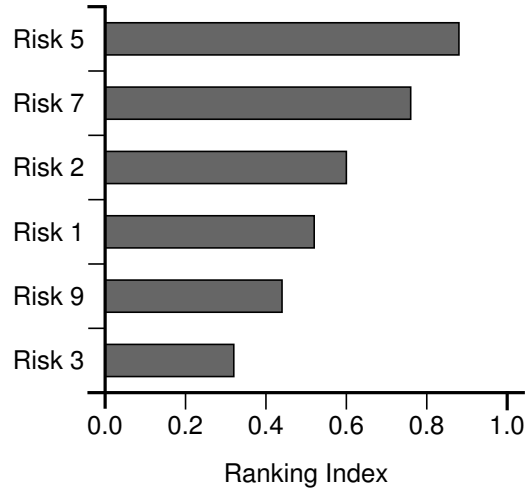
Figure 4: Ranked bar chart

In the remainder of this section, we provide an overview of the existing ranking indices and introduce two new ranking indices. The dynamics of these ranking indices are illustrated by means of an example.

4.1 Literature review

In what follows, we provide an overview of the existing ranking indices. For a more detailed discussion on the ranking indices presented below, refer to Elmaghraby (2000) and Demeulemeester and Herroelen (2002).

4.1.1 Critical Activities (CA)

A common practice in project risk management is to focus mitigation efforts on the critical activities of the deterministic early-start schedule $\mathfrak{s}$ (Goldratt 1997). The Critical Activities (CA) ranking values are computed as follows:

$$\mathrm{CA}_j^{(E)} = \delta_j, \tag{12}$$

where δ_j equals 1 if j is critical in $\mathfrak{s}$ and 0 otherwise.

While easy to implement, CA does not recognize the uncertain nature of a project. In addition, all activities on the critical chain have an equal ranking value, thereby severely limiting the discriminative power of the ranking index.

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4.1.2 Activity Criticality Index (*ACI*)

In stochastic CPM, the critical path is not fixed. For instance, the occurrence of risks may alter the critical path in a given network. The Activity Criticality Index (*ACI*) recognizes that almost any path and any activity can become critical with a certain probability (Van Slyke 1963). When using Monte Carlo simulation, the *ACI* of an activity is simply the proportion of simulation iterations during which the activity is critical:

$$\text{ACI}_j^{(E)} = \frac{1}{q} \sum_{p=1}^{q} \delta_{j,p}^{(E)}, \tag{13}$$

where $\delta_{j,p}^{(E)}$ equals 1 if j is critical in $\mathfrak{s}_p^{(E)}$ and 0 otherwise ($\mathfrak{s}_p^{(E)}$ is the early-start schedule during a simulation iteration p when activity durations are subject to a set of risks E).

ACI takes into account the criticality of an activity but not the variance of its duration. Therefore, ACI is unable to identify the activities that effectively contribute to the delay of the project. For instance, activities that are not impacted by risks (and therefore do not contribute to the delay of the project) can have a larger ACI than activities that become critical (and that contribute to the project delay) only when impacted by a risk.

4.1.3 Significance Index (*SI*)

The Significance Index (*SI*) was developed by Williams (1992) as an answer to criticism on *ACI*. When using Monte Carlo simulation, *SI* is computed as follows:

$$\text{SI}_j^{(E)} = \left(\frac{1}{\sum_{p=1}^{q} c_p^{(E)}} \right) \left[\sum_{p=1}^{q} \left(\frac{d_{j,p}^{(E)}}{d_{j,p}^{(E)} + \text{TF}_{j,p}^{(E)}} c_p^{(E)} \right) \right], \tag{14}$$

where $\text{TF}_{j,p}^{(E)}$ is the total float of an activity j during a simulation iteration p when activity durations are subject to a set of risks E (refer to Demeulemeester and Herroelen (2002) for a definition of total float).

SI tries to improve upon ACI by relating both the criticality of an activity and the project completion time. Similarly to ACI, however, SI does not take into account the variance of activity durations and is therefore also inherently flawed.

4.1.4 Cruciality Index (*CRI*)

The Cruciality Index (*CRI*) is defined as the absolute value of the correlation between the duration of an activity and the total project duration. When using Monte Carlo simulation, *CRI* is computed as follows:

$$\text{CRI}_j^{(E)} = \left| \text{corr}\left(\mathbf{d}_j^{(E)}, \mathbf{c}^{(E)} \right) \right|. \tag{15}$$

Although very intuitive, *CRI* has a few major drawbacks. First, it measures the linear relationship between the duration of an activity and the completion time of a project. As is well known, the relationship between these two entities may not be linear at all (Elmaghraby 2000). Second, *CRI* does not take into account the criticality of the activities themselves. For instance, a critical activity that has a small duration variability may have a smaller *CRI* than an activity that is not critical at all but that has a large duration variability.

4.1.5 Spearman Rank Correlation (*SRCA*)

Cho and Yum (1997) have criticized *CRI* because it assumes a linear relationship between the duration of an activity and the project completion time. They propose the use of a non-linear correlation measure such as the Spearman rank correlation coefficient. The Spearman Rank Correlation Index (*SRCA*) is computed as follows:

$$\text{SRCA}_j^{(E)} = \left| \text{corr}\left(\text{rank}\left(\mathbf{d}_j^{(E)} \right), \text{rank}\left(\mathbf{c}^{(E)} \right) \right) \right|. \tag{16}$$

SRCA improves upon *CRI* as it allows for monotonic relationships rather than linear relationships. Similarly to *CRI*, however, *SRCA* does not take into account the criticality of the activities and as such is also apt to produce counter-intuitive results.

4.1.6 Schedule Sensitivity Index (*SSI*)

The PMI Body of Knowledge (2008) and Vanhoucke (2010) define a ranking index that combines *ACI* and the variance of $\mathbf{d}_j^{(E)}$ and $\mathbf{c}^{(E)}$. When using Monte Carlo simulation, the Schedule Sensitivity Index (*SSI*) is computed as follows:

$$\text{SSI}_j^{(E)} = \text{ACI}_j^{(E)} \sqrt{\frac{\text{Var}\left(\mathbf{d}_j^{(E)} \right)}{\text{Var}\left(\mathbf{c}^{(E)} \right)}}. \tag{17}$$

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While *SSI* captures the variance of the activity durations as well as the variance of the project completion time, it ignores the covariance that might exist between these two entities.

4.1.7 Risk-Driven Ranking Indices

All prior ranking indices have been criticized in the literature (refer to Williams (1992), Elmaghraby (2000) and Cui et al. (2006)) and are primarily designed to rank activities, not risks. In this section, we introduce risk-driven ranking indices.

To the best of our knowledge, Hulett (2009) is the only reference that explicitly refers to a risk-driven ranking index. He proposes a simple adaptation of *CRI* such that it calculates the absolute value of the correlation between the impact of a risk and the project completion time. When using Monte Carlo simulation, the Cruciality Index for Risks (*CRIR*) is computed as follows:

$$\mathrm{CRIR}_e^{(E)} = \left| \mathrm{corr} \left(\mathbf{m}_e, \mathbf{c}^{(E)} \right) \right|, \tag{18}$$

where $\mathbf{m}_e = \{m_{e,1}, m_{e,2}, \ldots, m_{e,q}\}$, $\left(m_{e,p} = \sum_{j \in N} m_{j,e,p} \right)$ and $e \in E$. A similar adaptation may be made with respect to *SRCA*:

$$\mathrm{SRCR}_e^{(E)} = \left| \mathrm{corr} \left(\mathrm{rank} \left(\mathbf{m}_e \right), \mathrm{rank} \left(\mathbf{c}^{(E)} \right) \right) \right|. \tag{19}$$

No simple risk-driven adaptation exists for the other activity-based ranking indices (i.e., *CA*, *ACI*, *SI* and *SSI*).

4.2 Two new ranking indices

The aim of the new ranking indices is to redistribute the project delay over the combinations of activities and risks that cause the delay. More formally, the Critical Delay Contribution (*CDC*) of an activity j and a risk e may be

expressed as follows:

$$\text{CDC}_{j,e}^{(E)} \quad = \quad \frac{1}{q} \frac{\sum\limits_{p=1}^{q} m_{j,e,p} \delta_{j,p}^{(E)} \left(c_p^{(E)} - c \right)}{\sum\limits_{j \in N} \sum\limits_{e \in E} \sum\limits_{p=1}^{q} m_{j,e,p} \delta_{j,p}^{(E)}}, \tag{20}$$

$$= \quad E \left[\frac{\mathbf{m}_{j,e} \mathbf{y}_j^{(E)}}{\sum\limits_{j \in N} \sum\limits_{e \in E} \mathbf{m}_{j,e} \mathbf{y}_j^{(E)}} \right] \Delta^{(E)}, \tag{21}$$

where $\mathbf{y}_j^{(E)} = \left\{ \delta_{j,1}^{(E)}, \delta_{j,2}^{(E)}, \ldots, \delta_{j,q}^{(E)} \right\}$. $\text{CDC}_{j,e}^{(E)}$ represents the proportion of the project delay that originates from the impact of a risk $e : e \in E$ on an activity j.

From $\text{CDC}_{j,e}^{(E)}$, it is easy to obtain both an activity-based as well as a risk-driven ranking index:

$$\text{CDCA}_j^{(E)} \quad = \quad \sum_{e \in E} \text{CDC}_{j,e}^{(E)}, \tag{22}$$

$$\text{CDCR}_e^{(E)} \quad = \quad \sum_{j \in N} \text{CDC}_{j,e}^{(E)}, \tag{23}$$

where *CDCA* ranks activities and *CDCR* ranks risks.

4.3 Example

This section provides an example that allows us to illustrate the dynamics of the different ranking indices. After presenting the example data, we show how to compute the ranking values for each of the existing activity-based and risk-driven ranking indices. Next, we explain how to obtain the ranking values of the newly proposed ranking indices *CDCA* and *CDCR*.

4.3.1 Example data

Consider the project and corresponding early-start schedule that are presented in figure 5. The start and the completion of the project are represented by dummy activities 1 and 6 respectively. The non-dummy activities have deterministic durations $(d_2 = 1)$, $(d_3 = 2)$, $(d_4 = 4)$ and $(d_5 = 1)$. Therefore,

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EXAMPLE PROJECT NETWORK

DETERMINISTIC EARLY-START SCHEDULE
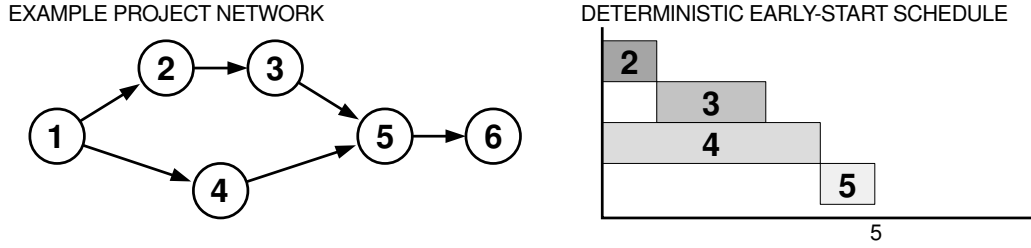
Figure 5: Example project network and corresponding early-start schedule

$E[M_{j,e}]$	$(e = X)$	$(e = Y)$	$(e = Z)$	Total
$(j = 2)$	0.50	1.50	0	2.00
$(j = 3)$	0.50	0	0	0.50
$(j = 4)$	0.50	0	1.25	1.75
$(j = 5)$	0	0	0	0
Total	1.50	1.50	1.25	4.25

Table 1: Example expected risk impacts

the deterministic project completion time is $(c = 5)$. Three risks have been identified. Let $E = \{X, Y, Z\}$ denote the set of risks that impact the activities of the example project. The expected risk impacts are given in table 1 (we assume that dummy activities are not impacted by risks).

In the example, we observe four simulation iterations (i.e., we assume that q equals four). For each simulation iteration p, figure 6 presents the GANTT chart of the early-start schedule of the non-dummy activities. Note that the expected risk impacts equal the average risk impacts over all simulation iterations.

For each simulation iteration p, table 2 presents:

- the duration of the non-dummy activities,

- the rank of the duration of the non-dummy activities,

- the criticality of the non-dummy activities,

- the total float of the non-dummy activities,

- the total risk impacts,

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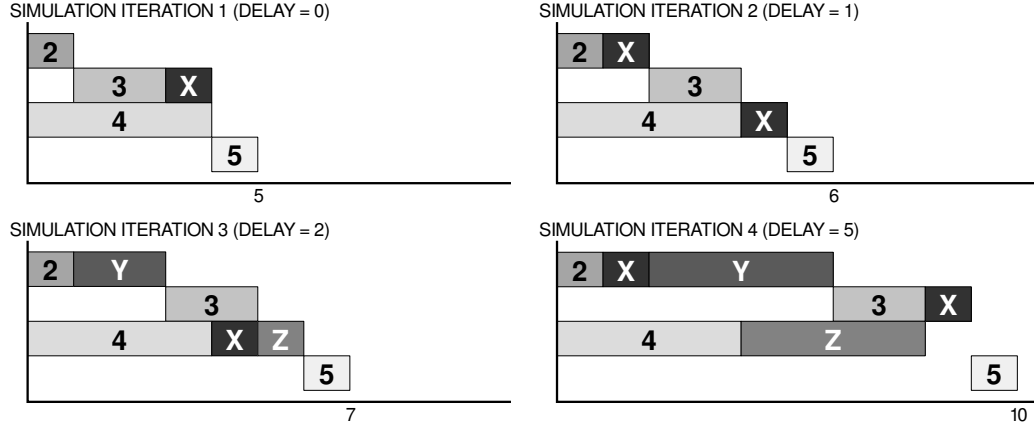
Figure 6: GANTT charts of the early-start schedule during the example simulation iterations

- the rank of the total risk impacts,

- the project completion time,

- the rank of the project completion time,

- the project completion time after risk mitigation (we assume that a risk is eliminated after being selected for mitigation)

As such, table 2 holds all the data required to compute the ranking values of the ranking indices discussed in the previous section. In addition, table 2 already shows us which risk should in fact be mitigated. More precisely, mitigating risk Y or risk Z results in an expected project completion time of 6.75 time units (i.e., a reduction of 0.25 time units). The mitigation of risk X on the other hand, yields a reduction of 0.75 time units. Risk X should therefore be selected as the risk on which to focus our mitigation efforts. In the upcoming sections, we discover whether the ranking indices are able to arrive at the same conclusion.

4.3.2 Activity-based ranking indices

Using the data from table 2, the ranking values of the different ranking indices can easily be computed. Table 3 presents the ranking values of the

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Activity durations						
$d_{j,p}^{(E)}$	$(p=1)$	$(p=2)$	$(p=3)$	$(p=4)$	Average	Variance
$d_{2,p}^{(E)}$	1	2	3	6	3.00	4.67
$d_{3,p}^{(E)}$	3	2	2	3	2.50	0.33
$d_{4,p}^{(E)}$	4	5	6	8	5.75	2.92
$d_{5,p}^{(E)}$	1	1	1	1	1.00	0.00

Rank of the activity durations						
rank $\left(d_{j,p}^{(E)}\right)$	$(p=1)$	$(p=2)$	$(p=3)$	$(p=4)$	Average	Variance
rank $\left(d_{2,p}^{(E)}\right)$	4	3	2	1	2.50	1.67
rank $\left(d_{3,p}^{(E)}\right)$	1	2	2	1	1.50	0.33
rank $\left(d_{4,p}^{(E)}\right)$	4	3	2	1	2.50	1.67
rank $\left(d_{5,p}^{(E)}\right)$	1	1	1	1	1.00	0.00

Activity criticality						
$\delta_{j,p}^{(E)}$	$(p=1)$	$(p=2)$	$(p=3)$	$(p=4)$	Average	Variance
$\delta_{2,p}^{(E)}$	1	0	0	1	0.50	0.33
$\delta_{3,p}^{(E)}$	1	0	0	1	0.50	0.33
$\delta_{4,p}^{(E)}$	1	1	1	0	0.75	0.25
$\delta_{5,p}^{(E)}$	1	1	1	1	1.00	0.00

Activity total float						
$TF_{j,p}^{(E)}$	$(p=1)$	$(p=2)$	$(p=3)$	$(p=4)$	Average	Variance
$TF_{2,p}^{(E)}$	0	1	1	0	0.50	0.33
$TF_{3,p}^{(E)}$	0	1	1	0	0.50	0.33
$TF_{4,p}^{(E)}$	0	0	0	1	0.25	0.25
$TF_{5,p}^{(E)}$	0	0	0	0	0.00	0.00

Total risk impact						
$m_{e,p}$	$(p=1)$	$(p=2)$	$(p=3)$	$(p=4)$	Average	Variance
$m_{X,p}$	1	2	1	2	1.50	0.33
$m_{Y,p}$	0	0	2	4	1.50	3.67
$m_{Z,p}$	0	0	1	4	1.25	3.58

Rank of the total risk impact						
rank $\left(m_{e,p}\right)$	$(p=1)$	$(p=2)$	$(p=3)$	$(p=4)$	Average	Variance
rank $\left(m_{X,p}\right)$	2	1	2	1	1.50	0.33
rank $\left(m_{Y,p}\right)$	3	3	2	1	2.25	0.92
rank $\left(m_{Z,p}\right)$	3	3	2	1	2.25	0.92

Project completion time						
	$(p=1)$	$(p=2)$	$(p=3)$	$(p=4)$	Average	Variance
$c_p^{(E)}$	5	6	7	10	7	4.67

Rank of the project completion time						
	$(p=1)$	$(p=2)$	$(p=3)$	$(p=4)$	Average	Variance
rank $\left(c_p^{(E)}\right)$	4	3	2	1	2.5	1.67

Project completion time after risk mitigation						
$c_p^{(E\backslash e)}$	$(p=1)$	$(p=2)$	$(p=3)$	$(p=4)$	Average	Variance
$c_p^{(E\backslash X)}$	5	5	6	9	6.25	3.58
$c_p^{(E\backslash Y)}$	5	6	7	9	6.75	2.92
$c_p^{(E\backslash Z)}$	5	6	6	10	6.75	4.92

Table 2: Example simulation output

Ranking values for the activity-based ranking indices				
$(\cdot)_j$	$(j=2)$	$(j=3)$	$(j=4)$	$(j=5)$
$\mathrm{CA}_j^{(E)}$	0	0	1.000	1.000
$\mathrm{ACI}_j^{(E)}$	0.500	0.500	0.750	1.000
$\mathrm{SI}_j^{(E)}$	0.866	0.845	0.960	1.000
$\mathrm{CRI}_j^{(E)}$	1.000	0.267	0.994	0
$\mathrm{SRCA}_j^{(E)}$	1.000	0	1.000	0
$\mathrm{SSI}_j^{(E)}$	0.500	0.134	0.593	0

Table 3: Example ranking values for activity-based ranking indices

activity-based ranking indices ($CDCA$ is not included here as it is discussed in an upcoming section).

When observing the ranking values of criticality-based ranking indices that do not take into account the variance of activity durations (i.e., CA, ACI and SI), it is clear that activity 5 is considered to have the largest impact on the delay of the project. Activity 5, however, is not at all impacted by risks, making it rather hard to determine the risk on which to focus our mitigation efforts (i.e., we are unable to identify the root causes of uncertainty). Two options arise: (1) we do not focus on activity 5 but select the highest-ranked activity that is still impacted by risks and (2) we select and mitigate a random risk. It is clear that the latter option is detrimental to the performance of the ranking index. In addition, it makes sense to assume that a project manager knows which activities are impacted by risks. Therefore, in this article, activity-based ranking indices will only rank activities that are impacted by risks.

When ranking only activities that are impacted by risks, we observe that the ranking values of all criticality-based ranking indices (i.e., CA, ACI, SI and SSI) select activity 4 as the most important activity. Activity 4 is impacted by two risks (i.e., risk X and risk Z), leaving us with two options: (1) we randomly select one of both risks and (2) we select the risk that has the largest expected impact on activity 4. In this article, we assume that a project manager knows the expected impact of each risk on each of the activities. Therefore, activity-based ranking indices select the risk that has the largest expected impact on the highest-ranked activity. As a result, all criticality-based ranking indices select risk Z as the best risk.

$(\cdot)_e$	$(e = X)$	$(e = Y)$	$(e = Z)$
$\mathrm{CRIR}_j^{(E)}$	0.535	0.967	0.978
$\mathrm{SRCR}_j^{(E)}$	0.447	0.944	0.944

Table 4: Example ranking values for risk-driven ranking indices

With respect to ranking index *CRI*, activity 2 is ranked highest. Similarly to activity 4, activity 2 is impacted by two risks (risk X and risk Y) of whom risk Y has the largest expected impact. Therefore, *CRI* selects risk Y to be mitigated. *SRCA*, however, assigns the highest ranking value to both activities 2 and 4. As such, we have to decide to either focus our mitigation efforts on activity 2 (i.e., mitigate risk Y) or on activity 4 (i.e., mitigate risk Z). Because only one risk can be mitigated at a time, it makes sense to select the risk that has the largest expected impact (i.e., from all risks impacting the highest-ranked activities, we select the one that has the largest expected impact). Therefore, ranking index *SRCA* selects risk Y to be mitigated (i.e., all correlation-based ranking indices choose to mitigate risk Y).

4.3.3 Risk-driven ranking indices

Table 4 presents the ranking values of the risk-driven ranking indices (*CDCR* is not included here as it is discussed in the upcoming section).

We observe that both *CRIR* and *SRCR* fail to select risk X as the best candidate for mitigation. *CRIR* indicates that risk Z should be mitigated whereas *SRCR* assigns an equal ranking value to both risk Y and risk Z. If multiple risks are ranked highest, one of them has to be chosen randomly. In order to evaluate the outcome of this random selection of risks, we observe two scenarios. In a first scenario, risk Y is mitigated, whereas in a second scenario risk Z is mitigated. Both scenarios have equal probability of being realized and as such have a weight of 50 percent. The mitigation of either risk Y or risk Z results in a project completion time of 6.75 time units. As a result, the project completion time after mitigation equals 6.75 time units when adopting ranking index *SRCR*.

	RISK IMPACT ON DURATION				CONTRIBUTION TO DELAY				PROPORTION OF CDC				
	RISK X	RISK Y	RISK Z	TOTAL	RISK X	RISK Y	RISK Z	TOTAL	RISK X	RISK Y	RISK Z	TOTAL	
SIMULATION ITERATION 1 (DELAY = 0)													
2	0	0	0	0	0	0	0	0	0	0	0	0	
3 X	1	0	0	1	0	0	0	0	0	0	0	0	
4	0	0	0	0	0	0	0	0	0	0	0	0	
5	0	0	0	0	0	0	0	0	0	0	0	0	
TOTAL (5)	1	0	0	1	0	0	0	0	0	0	0	0	
SIMULATION ITERATION 2 (DELAY = 1)													
2 X	1	0	0	1	0	0	0	0	0	0	0	0	
3	0	0	0	0	0	0	0	0	0	0	0	0	
4 X	1	0	0	1	1	0	0	1	0.25	0	0	0.25	
5	0	0	0	0	0	0	0	0	0	0	0	0	
TOTAL (6)	2	0	0	2	1	0	0	1	0.25	0	0	0.25	
SIMULATION ITERATION 3 (DELAY = 2)													
2 Y	0	2	0	2	0	0	0	0	0	0	0	0	
3	0	0	0	0	0	0	0	0	0	0	0	0	
4 X Z	1	0	1	2	1	0	1	2	0.25	0	0.25	0.5	
5	0	0	0	0	0	0	0	0	0	0	0	0	
TOTAL (7)	1	2	1	4	1	0	1	2	0.25	0	0.25	0.5	CDCA
SIMULATION ITERATION 4 (DELAY = 5)													
2 X Y	1	4	0	5	0.833	3.333	0	4.167	0.208	0.833	0	1.042	1.042
3 X	1	0	0	1	0.833	0	0	0.833	0.208	0	0	0.208	0.208
4 Z	0	0	4	4	0	0	0	0	0	0	0	0	0.75
5	0	0	0	0	0	0	0	0	0	0	0	0	0
TOTAL (10)	2	4	4	10	1.667	3.333	0	5	0.417	0.833	0	1.25	2
CDCR									0.917	0.833	0.25	2	

Figure 7: Illustration of the computation of *CDCA* and *CDCR*

4.3.4 Computation of *CDCA* and *CDCR*

Figure 7 illustrates the computation of the ranking values of *CDCA* and *CDCR*. For each simulation iteration p, figure 7 presents: (1) a GANTT chart of the early-start schedule of the non-dummy activities, (2) the impact of a risk e on the duration of an activity j, (3) the proportion of the project delay that is contributed by each of the activity-risk combinations and (4) the proportion of *CDC* that can be assigned to each of the activity-risk combinations.

In a first simulation iteration, risk X impacts activity 3. The project, however, is not delayed and as such there is no delay to be distributed among the activity-risk combinations.

In a second simulation iteration, risk X impacts both activities 2 and 4. As a result, the project is delayed by 1 time unit. It is clear that in this simulation iteration, activity 2 is not critical. Therefore, activity 2 does not contribute to the delay of the project even though it is impacted by risk X. Activity 4 on the other hand, is critical and is also impacted by risk X. Therefore, activity 4 (as well as risk X) contributes 1 time unit to the delay of the project. Because we observe four simulation iterations, the proportion of CDC that can be assigned to activity-risk combination $(4, X)$ equals $1/4$ time unit.

In a third simulation iteration, risk X again impacts activity 4. In addition, risk Z also impacts activity 4 whereas activity 2 is impacted by risk Y. The project delay equals 2 time units. In this simulation iteration, activity 4 is the only impacted activity that is critical. As such, activity 4 is assigned the 2 time units of project delay. In order to distribute these 2 time units of project delay over the root risks, we use the risk impacts as weights. Therefore, risk X and risk Z are both assigned 1 time unit of project delay. Dividing by the number of simulation iterations, yields the proportion of CDC that can be assigned to the respective activity-risk combinations (i.e., $1/4$ time unit for both activity-risk combinations $(4, X)$ and $(4, Z)$).

In a fourth and final simulation iteration, risk X impacts activities 2, 3 and 4. Risk Y impacts activity 2 and risk Z impacts activity 4. The project is delayed by 5 time units. In this simulation iteration, both activities 2 and 3 are critical and their duration is impacted by risks X and Y (activity 4 is not critical and as such does not contribute to the delay of the project). In order to distribute the delay of the project over the critical activities, we use the impact of all risks on their duration as weights. Activity 2 incurs a total risk impact of 5 time units whereas the impact on the duration of activity 3 equals 1 time unit. Therefore, activities 2 and 3 are assigned $25/6$ and $5/6$ time units of project delay respectively. In order to distribute the project delay over the root risks, we once more use the risk impacts themselves as weights. The impact on the duration of all critical activities equals 6 time units. Therefore, one third of the project delay is assigned to risk X whereas risk Y is assigned the remaining two thirds. Dividing by the number of simulation iterations yields the proportion of CDC that can be assigned to each of the activity-risk combinations (i.e., $5/24$ time units for both activity-risk combinations $(2, X)$ and $(3, X)$ and $5/6$ time units for activity-risk combination $(2, Y)$).

When summing over all simulation iterations we obtain the ranking values of ranking indices $CDCA$ and $CDCR$ (note that the expected project delay

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$\Delta^{(E)}$ equals the sum of all ranking values for $CDCA$ as well as for $CDCR$). We observe that $CDCA$ considers activity 2 to be the activity that contributes most to the delay of the project. Therefore, $CDCA$ selects risk Y (i.e., the risk that has the largest expected impact on the duration of activity 2) to be mitigated. $CDCR$, however, assigns the highest ranking value to risk X, making it the only ranking index that is able to correctly identify the best candidate for mitigation.

5 Computational Experiment

Contrary to most of the literature, we will not assess the performance of ranking indices by means of counterexamples. Our goal is to evaluate the resilience of ranking indices in a wide variety of settings, using an extensive experimental design. At the core of the experiment are the PSPLIB J120 project networks (Kolisch and Sprecher 1996). For each of these networks and for each of the 48 distinct risk profiles defined below, we will evaluate the mitigation efficiency of the ranking indices discussed in the previous section. A similar approach is followed in Vanhoucke (2010), who considers only activity-based ranking indices.

In what follows, we first discuss the experimental design itself. Next, we deal with the experimental setup and discuss the main results. Finally, we evaluate the validity and accuracy of the simulation model that is used to assess the performance of the ranking indices.

5.1 Experimental design

For each of the projects in the PSPLIB J120 data set, uncertainty is introduced by modeling a number of risks. Five parameters were selected to characterize the risks: (1) risk uniformity, (2) risk quantity, (3) risk probability, (4) risk impact and (5) risk correlation. The settings of these parameters are based on our experience in the risk management field.

Risk uniformity deals with the number of activities that are impacted by a single risk. Often, clusters of activities have a similar task content and hence are subject to similar risks. We refer to these clusters of activities as activity groups (Schatteman et al. 2008). When risk uniformity is low, the number of activities impacted by any risk $e \in R$ follows a discrete uniform distribution with minimum and maximum equal to 1 and 3 activities respectively. A

low risk uniformity setting results in an average of 60 activity groups in a project network. The average number of activities in an activity group equals 2. When risk uniformity is high, the number of activities impacted by any risk $e \in R$ follows a discrete uniform distribution with minimum and maximum equal to 1 and 11 activities respectively. A high risk uniformity setting corresponds to an average of 20 activity groups in a project network whereas the average number of activities in an activity group equals 6.

Risk quantity indicates the number of risks that are identified during the risk identification process. A low risk quantity setting corresponds to a project in which activities are impacted by 25 risks. When risk quantity is high, 50 risks impact the activities of a project. Risks are randomly assigned to a single activity group. Note that it is possible that some activity groups (and hence activities) are not impacted by risks (e.g., if the number of risks is smaller than the number of activity groups).

Risk probability indicates the probability of occurrence of a risk whereas *risk impact* defines the impact of a risk on the duration of an activity. We define two types of risks: (1) risks with a large impact but with a small probability of occurrence and (2) risks with a small impact but with a large probability of occurrence. Risks are randomly assigned to a risk type, where each risk has a 25 percent chance of being of type 1 (as such, risks have a 75 percent chance of being of type 2). For both risk types, we allow for high and low settings of risk probability and risk impact. Table 5 presents the adopted parameter settings. Note that: (1) the impact of a risk is modeled as a fixed extension of the duration of an activity and follows a triangular distribution and (2) the risk probability is modeled using a continuous uniform distribution. We opt for the use of the triangular and the uniform distribution as our experience learns that project managers find it easier to assess the parameters that correspond to these distributions (e.g., a project manager is able to assess the worst, best and most likely impact of a risk rather than the alpha and beta parameter of a beta distribution).

Risk correlation indicates whether the occurrences of a risk (on activities in the impacted activity group) are correlated. We investigate three possible scenarios. A first scenario deals with the setting in which there is perfect correlation (i.e., either all activities in the activity group are impacted or none are). The second scenario, assumes that risk occurrences are independent (i.e., there is no correlation between risk occurrences). In a third scenario, we assume that the risk correlation is random, indicating that the occurrences of a risk are correlated with a random correlation factor that is drawn from

Risk probability	Risk impact	Risk type	Probability		Impact		
			min	max	min	most likely	max
High	High	Type 1	0.05	0.05	1.0	2.0	9.0
		Type 2	0.1	0.7	0.0	1.0	2.0
High	Low	Type 1	0.05	0.05	0.5	1.0	4.5
		Type 2	0.1	0.7	0.0	0.5	1.0
Low	High	Type 1	0.025	0.025	1.0	2.0	9.0
		Type 2	0.05	0.35	0.0	1.0	2.0
Low	Low	Type 1	0.025	0.025	0.5	1.0	4.5
		Type 2	0.05	0.35	0.0	0.5	1.0

Table 5: Parameter settings for risk probability and risk impact

a continuous uniform distribution with minimum and maximum equal to 0 and 1 respectively.

The possible settings of the five parameters combine to 48 distinct risk profiles that are to be evaluated. For each risk profile and over all project networks in the PSPLIB J120 data set, we will evaluate and compare the mitigation efficiency of the ranking indices. Note that in this experiment, we assume that the mitigation of a risk results in the elimination of that risk.

5.2 Experimental setup

In the experiment, we evaluate a total of 12 ranking indices, namely the ten ranking indices discussed in section 4 (*CA*, *ACI*, *SI*, *CRI*, *SRCA*, *SSI*, *CRIR*, *SRCR*, *CDCA* and *CDCR*) as well as two additional ranking indices: (1) *RAND* randomly selects a risk from those risks still active and may be considered as a worst-case scenario and (2) *OPT* is a greedy-optimal procedure that evaluates the elimination of each risk and that selects the risk that yields the largest reduction in project delay. *OPT* may be considered as a best-case scenario, but has limited practical value due to its computational requirements.

In what follows, we demonstrate how to obtain the set of highest-ranked risks for both activity-based and risk-driven ranking indices. Next, we define two performance measures and introduce a stepwise procedure that may be used to assess the performance of a ranking index for a given project and a given risk profile. In addition, we present the outline of the computational experiment that allows us to evaluate the performance of all ranking indices

over all projects and all risk profiles.

5.2.1 Determining the set of highest-ranked risks

Let $B^{(E(\cdot))}$ denote the subset of E that contains all risks in E that are ranked highest by ranking index $(\cdot)$. If $(\cdot)$ is a risk-driven ranking index, $B^{(E(\cdot))}$ is defined as follows:

$$B^{(E(\cdot))} = \left\{ e^* : (\cdot)_{e^*} = \max\left((\cdot)_e\right) \forall e^* : e \in E \right\}. \tag{24}$$

If $(\cdot)$ is an activity-based ranking index, a two-step procedure is required in order to obtain the set of highest-ranked risks. In a first step, we determine $A^{(N(\cdot))}$, the subset of N that contains all activities in N that are ranked highest by ranking index $(\cdot)$. $A^{(N(\cdot))}$ is defined as follows:

$$A^{(N(\cdot))} = \left\{ j^* : (\cdot)_{j^*} = \max\left((\cdot)_j\right) \forall j^*, j \in N \right\}. \tag{25}$$

In a second step, we select the risks that have the largest expected impact on the highest-ranked activities:

$$B^{(E(\cdot))} = \left\{ e^* : E\left[M_{j,e^*}\right] = \max\left(E\left[M_{j,e}\right]\right) \forall j \in A^{(N(\cdot))} \wedge \forall e^*, e \in E \right\}. \tag{26}$$

The two-step procedure assumes that all information known to the risk-driven ranking indices is also known to the activity-based ranking indices (i.e., the decision maker knows: (1) which activities are impacted by risks and (2) the expected impact of each risk on each of the activities). Therefore, the comparison between activity-based and risk-driven ranking indices is made as fair as possible.

5.2.2 Definition of performance measures

In order to compare the performance of the ranking indices, define the Relative Residual Delay after mitigation of x risks using ranking index $(\cdot)$:

$$\text{RRD}^{(\cdot)_x} = \frac{\Delta^{(\cdot)_x}}{\Delta^{(\cdot)_0}}, \tag{27}$$

where $\Delta^{(\cdot)_x}$ is the expected project delay after mitigation of x risks using ranking index $(\cdot)$ and $\Delta^{(\cdot)_0}$ is the expected project delay before any mitigation

takes place. It is clear that a smaller value for $\mathrm{RRD}^{(\cdot)_x}$ corresponds to a more effective ranking index.

Another measure to assess the performance of a ranking index $(\cdot)$ is the Mitigation Efficiency Index $(\mathrm{MEI}^{(\cdot)})$. $\mathrm{MEI}^{(\cdot)}$ is defined as follows:

$$\mathrm{MEI}^{(\cdot)} = 1 - 2\frac{\sum\limits_{x=1}^{r} \mathrm{RRD}^{(\cdot)_x}}{r - 1} \tag{28}$$

The details of the dynamics of this measure may be found in the appendix. In short, $\mathrm{MEI}^{(\cdot)}$ is supported on the $[-1, 1]$ real interval, where a value of $(\mathrm{MEI}^{(\cdot)} = 0)$ indicates that the performance of the ranking index equals that of the random procedure. A value of $(\mathrm{MEI}^{(\cdot)} = 1)$ on the other hand, refers to the optimal case in which mitigating a single risk is sufficient to resolve all project uncertainty. It is clear that it is impossible to attain a value of $(\mathrm{MEI}^{(\cdot)} = 1)$ in general.

5.2.3 Assessing the performance of a ranking index $(\cdot)$

For a given risk profile and a given project in the PSPLIB J120 data set, we generate $\mathbf{m}_{j,e}$, the vector of common risk impacts for all risks $e : e \in R$ and for all activities $j : j \in N$. Using these risk impacts, we assess the mitigation efficiency of a ranking index $(\cdot)$ by means of a stepwise procedure.

In each step t, we evaluate a scenario $\pi_t : \pi_t \in \Pi$ (where Π is the set of all scenarios). A scenario π_t is fully characterized by: (1) a weight λ_t and (2) a set of risks $E_t : E_t \subseteq R$. Given a set of risks E_t and the common risk impacts generated earlier, we use equation 7 in order to obtain the vector of realized durations $\mathbf{d}_j^{(E_t)}$ for all $j : j \in N$. In turn, $\mathbf{d}_n^{(E_t)}$ may be used to calculate the expected project delay $\Delta^{(E_t)}$ using equations 8 – 11. During each of the steps in the stepwise procedure, we update $\Delta^{(\cdot)_x}$ as follows (note that $\Delta^{(\cdot)_x}$ is initialized to zero for all $x : x \in \{0, 1, \ldots, r\}$ at the start of the stepwise procedure):

$$\Delta^{(\cdot)}\big|^{E_t^{\complement}}\big| = \Delta^{(\cdot)}\big|^{E_t^{\complement}}\big| + \left(\lambda_t \Delta^{(E_t)}\right), \tag{29}$$

where $E_t^{\complement}$ is the complement of E in R. Next, we use ranking index $(\cdot)$ to obtain $B^{(E_t(\cdot))}$, the set of highest-ranked risks. For each of the best risks included in $B^{(E_t(\cdot))}$, we determine the parameters of a scenario π_{u_e}. More

formally, for all $e : e \in B^{(E_t(\cdot))}$, we define a scenario π_{u_e} that is fully characterized by:

$$\lambda_{u_e} = \frac{\lambda_t}{|B^{(E_t(\cdot))}|}, \tag{30}$$

$$E_{u_e} = E_t \setminus e. \tag{31}$$

If $(E_{u_e} = E_z)$ for any $z : \pi_z \in \Pi$, the scenario already exists. If the scenario already exists, the weight of the existing scenario is increased as follows:

$$\lambda_z = \lambda z + \lambda u_e. \tag{32}$$

If the scenario does not yet exist, we add it to the set of all scenarios Π (i.e., $(\Pi = \Pi \cup \{\pi_{u_e}\})$). After all risks in $B^{(E_t(\cdot))}$ have been processed, we proceed to step $(t+1)$ and repeat the procedure until all scenarios in Π have been evaluated.

The stepwise procedure starts with the evaluation of scenario π_1 (π_1 is characterized by a weight $(\lambda_1 = 1)$ and a set of risks $(E_1 = R)$). The stepwise procedure ends after evaluation of scenario $\pi_{|\Pi|}$ ($\pi_{|\Pi|}$ is characterized by a weight $(\lambda_{|\Pi|} = 1)$ and a set of risks $(E_{|\Pi|} = \emptyset)$). An outline of the procedure is given in algorithm 1. In addition, Figure 8 further illustrates the dynamics of the stepwise procedure.

5.2.4 Computational experiment

The performance of the different ranking indices is assessed: (1) for each of the 600 projects in the PSPLIB J120 data-set, (2) for each of the 48 risk profiles and (3) for each step in the mitigation process. An outline of the computational experiment is given in algorithm 2.

The computational experiment is coded in Visual C++ and was executed on a Pentium IV 2.67 GHz personal computer. To obtain statistically significant results, the execution of each scenario is simulated 1000 times. To further increase the precision of the simulation results, we adopt variance reduction techniques. More specifically, we use common random numbers when assessing the mitigation efficiency of the different ranking indices for a given project and a given risk profile (i.e., during each step of the mitigation process and for each of the ranking indices, the same set of risk impacts is used). The validity and accuracy of the simulation model are discussed in section 5.4.

Algorithm 1 Stepwise procedure

---

for $x = 0$ to r **do**
    Initialize $\Delta^{(\cdot)}{}_x$ to zero
end for
Define a first scenario π_1 characterized by parameters $\lambda_1 = 1$ and $E_1 = R$
for all scenarios $\pi_t \in \Pi$ **do**
    for $p = 1$ to q **do**
        for $j = 1$ to n **do**
            Use common risk impacts $m_{j,e,p}$ to compute $d_{j,p}^{(E_t)}$ using equation 7
        end for
    end for
    Use $\mathbf{d}_n^{(E_t)}$ to calculate $\Delta^{(E_t)}$ using equations $8 - 11$
    Update $\Delta^{(\cdot)}_{\left|E_t^{\mathbb{C}}\right|}$ using equation 29
    if $(\cdot)$ is activity-based **then**
        Obtain $A^{(N(\cdot))}$ using equation 25
        From $A^{(N(\cdot))}$, obtain $B^{(E_t(\cdot))}$ using equation 26
    else
        Obtain $B^{(E_t(\cdot))}$ using equation 24
    end if
    for all $e \in B^{(E_t(\cdot))}$ **do**
        Determine the parameters of scenario π_{u_e} using equations $30 - 31$
        if $E_{u_e} = E_z$ for any $z : \pi_z \in \Pi$ **then**
            Adapt the weight of π_z using equation 32
        else
            Add π_{u_e} to Π
        end if
    end for
end for

Algorithm 2 Computational experiment

---

for all Project networks in the PSPLIB J120 data set **do**
    for all Risk uniformity settings **do**
        Assign activities to activity groups
        for all Risk quantity settings **do**
            Set r and define $R = \{1, 2, \ldots, r\}$
            for all Risk probability settings **do**
                for all Risk impact settings **do**
                    for $e = 1$ to r **do**
                        Set the probability and impact of each risk
                    end for
                    for all Risk correlation settings **do**
                        Set the correlation of risk occurrences
                        for $e = 1$ to r **do**
                            for $j = 1$ to n **do**
                                for $p = 1$ to q **do**
                                    For the current project and risk profile, generate common risk impact $m_{j,e,p}$
                              end for
                            end for
                        end for
                      for all Ranking indices $(\cdot)$ **do**
                        use algorithm 1 to obtain $\Delta^{(\cdot)}x$ for all $x : x \in \{0, 1, \ldots, r\}$
                        for all $x = 0$ to r **do**
                            compute performance measure $\text{RRD}^{(\cdot)}x$ using equation 27
                        end for
                        For the current project and the current risk profile, compute performance measure $\text{MEI}^{(\cdot)}$ using equation 28
                    end for
                  end for
              end for
            end for
        end for
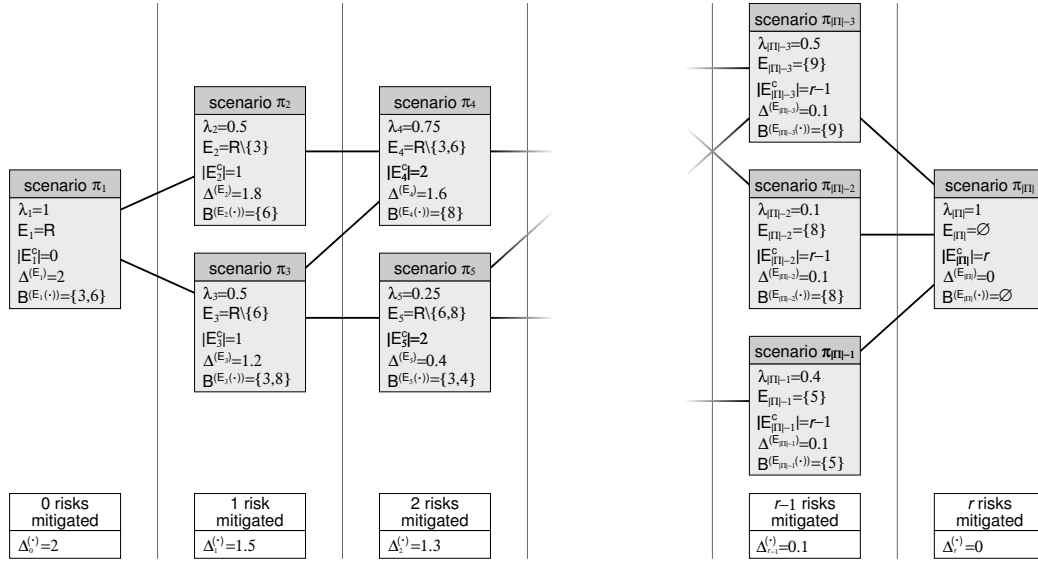   **end for**
end for

Figure 8: Illustration of the stepwise procedure that is used to assess ranking index performance

5.3 Results

Figure 9 gives an overview of the average performance of the activity-based ranking indices with respect to measure $\mathrm{RRD}^{(\cdot)_x}$ for the range starting from $(x = 0)$ until $(x = 10)$ (i.e., ten risks have been mitigated). The data are aggregated over all 600 project networks in the PSPLIB J120 data sets and over all 48 risk profiles. We observe that the mitigation of risks results in a decrease of the expected project delay for each ranking index. Because *RAND* randomly selects risks, its improvement is linear with the number of risks mitigated. All other ranking indices follow a convex curve, implying that risks with a larger impact on the project delay are selected first. One might conclude that *CDCA* is outperformed only by *SRCA*. It is clear, however, that there still exists a gap between the performance of the activity-based indices and the *OPT* procedure.

Similarly to figure 9, figure 10 presents the performance of risk-driven ranking indices with respect to measure $\mathrm{RRD}^{(\cdot)_x}$. We observe that *SRCR* outperforms *CRIR* as well as the activity-based ranking indices. Of larger importance, however, is the observation that *CDCR* (the risk-driven ranking
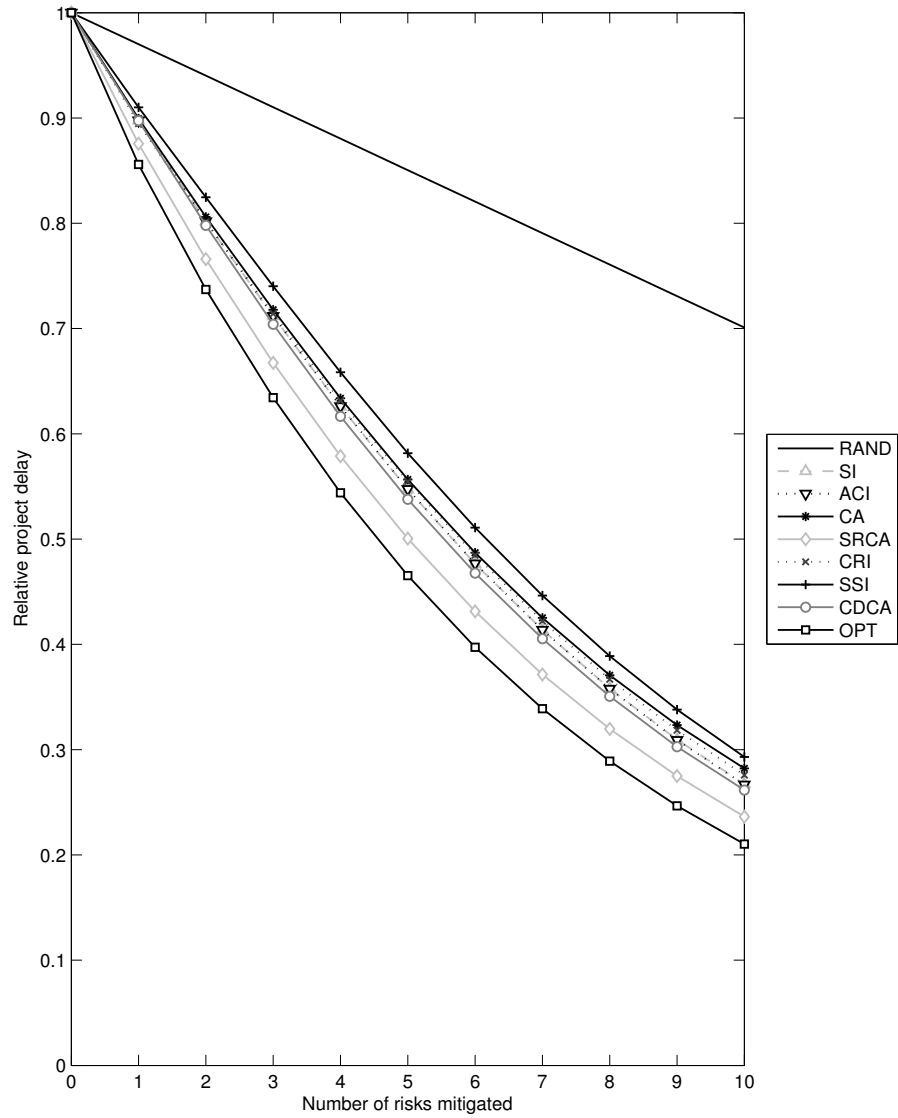
Figure 9: Mitigation efficiency of activity-based ranking indices
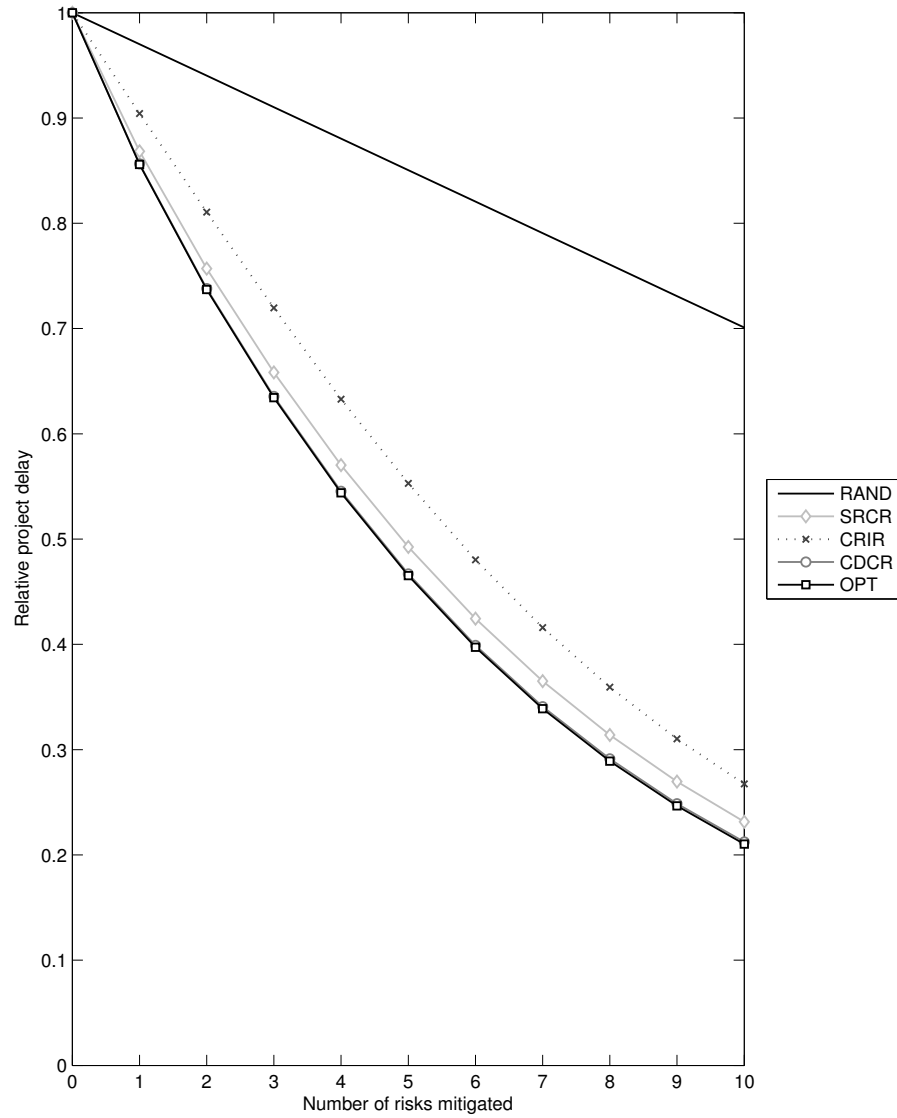
Figure 10: Mitigation efficiency of risk-driven ranking indices

index proposed in this article) easily outperforms *CRIR* and *SRCR* and even matches the performance of the *OPT* procedure. It is clear that *CDCR* sets a new standard in the field of ranking indices.

Table 6 presents the performance of the different ranking indices with respect to measure $\text{MEI}^{(\cdot)}$. We observe that $\text{MEI}^{(\text{RAND})}$ is smaller than 0.001 (i.e., is close to zero), indicating that the *RAND* procedure has no real mitigation potential. The *OPT* procedure boosts the highest values of $\text{MEI}^{(\cdot)}$ and is rivaled only by *CDCR*. Virtually no difference exists between the performance of the *OPT* procedure and the *CDCR* ranking index. With respect to the activity-based ranking indices, it is clear that *SRCA* takes the pole position closely followed by *CDCA*, *ACI* and *SI*.

Furthermore, we observe that risk correlation seems to have a limited impact on the performance of the ranking indices (certainly for those ranking indices that perform well). Even for correlation-based ranking indices (i.e., *CRI*, *CRIR*, *SRCA* and *SRCR*) the difference in performance is not very outspoken. A similar conclusion holds for risk probability. Its impact on the performance of ranking indices is subtle to non-existing. Risk uniformity on the other hand substantially affects the $\text{MEI}^{(\cdot)}$ of the different ranking indices. It is clear that a higher risk uniformity results in lower values of $\text{MEI}^{(\cdot)}$ (i.e., it is easier to distinguish between risks that only impact a small number of activities). With respect to risk quantity, we observe that the identification of more risks results in a decreased performance (i.e., if there are more risks, the mitigation of a single risk tends to be less effective). Risk impact has a negative effect on the $\text{MEI}^{(\cdot)}$ of a ranking index. Lower risk impacts correspond to higher values of $\text{MEI}^{(\cdot)}$ (i.e., the relative effect of mitigating a risk increases if there are only few risks that impact project objectives).

5.4 Model accuracy and ranking value convergence

In this section, we first discuss the number of simulation iterations required to obtain convergence in the project completion times. Next, we observe the convergence in ranking values for each of the ranking indices. For both tests, we observe only a single risk profile. More specifically, from the risk profiles defined in section 5.1, we select the profile that has the largest variability in project completion time (i.e., we assume a high risk uniformity, risk quantity, risk probability and risk impact). We assume that there is no risk correlation.

Index	Avg	Corr	MEI$^{(\cdot)}$															
			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	.000	0	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
RAND	.000	RND	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
	.000	1	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
	.698	0	.57	.60	.59	.61	.59	.61	.60	.62	.77	.79	.78	.80	.79	.81	.80	.82
OPT	.697	RND	.57	.60	.58	.61	.58	.61	.59	.62	.78	.80	.79	.80	.79	.81	.80	.82
	.695	1	.56	.59	.58	.60	.58	.60	.59	.61	.78	.80	.79	.80	.79	.81	.80	.82
	.621	0	.46	.50	.49	.52	.48	.52	.50	.53	.69	.75	.72	.76	.73	.77	.74	.78
CA	.619	RND	.45	.50	.48	.52	.48	.52	.50	.52	.70	.75	.72	.76	.73	.77	.74	.78
	.614	1	.44	.49	.47	.50	.47	.51	.49	.52	.70	.75	.72	.76	.73	.77	.74	.78
	.643	0	.49	.52	.51	.53	.51	.53	.52	.54	.74	.76	.75	.77	.77	.78	.77	.79
ACI	.640	RND	.49	.52	.50	.52	.50	.52	.51	.53	.75	.77	.76	.77	.76	.78	.77	.79
	.637	1	.48	.51	.50	.52	.50	.52	.51	.53	.75	.77	.75	.77	.76	.78	.77	.79
	.641	0	.49	.52	.51	.53	.51	.53	.52	.53	.74	.76	.75	.77	.76	.78	.77	.79
SI	.639	RND	.49	.52	.50	.52	.50	.52	.51	.53	.74	.77	.75	.77	.76	.78	.77	.79
	.637	1	.48	.51	.49	.52	.50	.52	.51	.53	.74	.77	.75	.77	.76	.78	.77	.79
	.612	0	.45	.49	.47	.50	.45	.49	.47	.50	.71	.74	.73	.75	.74	.76	.75	.77
CRI	.636	RND	.49	.53	.51	.55	.49	.52	.52	.54	.73	.75	.74	.76	.75	.77	.76	.77
	.643	1	.49	.53	.52	.55	.50	.54	.53	.56	.73	.76	.75	.77	.75	.77	.76	.78
	.657	0	.49	.53	.51	.54	.52	.55	.53	.56	.75	.78	.77	.79	.78	.81	.79	.81
SRCA	.677	RND	.53	.56	.55	.58	.55	.58	.56	.59	.76	.79	.78	.80	.79	.81	.79	.81
	.680	1	.53	.57	.55	.58	.56	.59	.57	.60	.77	.79	.78	.80	.79	.81	.80	.82
	.616	0	.46	.48	.48	.50	.46	.48	.48	.49	.73	.75	.74	.76	.75	.77	.76	.77
SSI	.614	RND	.45	.48	.48	.50	.46	.48	.47	.49	.73	.75	.74	.76	.75	.77	.76	.77
	.610	1	.44	.47	.46	.49	.45	.47	.47	.48	.73	.75	.74	.76	.75	.77	.75	.77
	.646	0	.47	.51	.49	.52	.50	.53	.51	.54	.75	.78	.76	.79	.78	.80	.79	.81
CDCA	.644	RND	.46	.51	.48	.53	.49	.53	.50	.54	.75	.78	.76	.79	.78	.80	.79	.81
	.640	1	.45	.50	.47	.51	.48	.52	.50	.53	.75	.78	.76	.79	.78	.80	.78	.81
	.639	0	.49	.53	.51	.54	.52	.55	.53	.55	.72	.75	.74	.76	.75	.77	.76	.78
CRIR	.638	RND	.49	.53	.52	.55	.50	.54	.53	.55	.73	.75	.74	.76	.75	.77	.76	.77
	.635	1	.48	.51	.51	.54	.50	.53	.53	.55	.73	.75	.74	.76	.75	.77	.76	.77
	.674	0	.52	.56	.54	.57	.56	.58	.57	.60	.75	.78	.77	.79	.78	.80	.80	.81
SRCR	.684	RND	.54	.58	.56	.60	.56	.59	.58	.60	.76	.78	.78	.80	.79	.81	.80	.82
	.676	1	.50	.54	.56	.59	.55	.58	.58	.60	.75	.78	.78	.80	.79	.81	.80	.82
	.697	0	.57	.60	.58	.61	.59	.61	.60	.62	.77	.79	.78	.80	.79	.81	.80	.82
CDCR	.695	RND	.56	.60	.58	.61	.58	.61	.59	.61	.77	.80	.78	.80	.79	.81	.80	.82
	.692	1	.56	.59	.57	.60	.57	.60	.59	.61	.78	.80	.78	.80	.79	.81	.80	.82
Risk uniformity			High								Low							
Risk quantity			High				Low				High				Low			
Risk probability			High		Low		High		Low		High		Low		High		Low	
Risk impact			H	L	H	L	H	L	H	L	H	L	H	L	H	L	H	L

Table 6: Mitigation efficiency of the different ranking indices

5.4.1 Convergence in project completion times

For a given project $l \in L$ (where L is the set of all projects in the PSPLIB J120 data set), $\mu_{l,v}^{(\cdot)_x}$ and $\sigma_{l,v}^{(\cdot)_x}$ denote the expected value and the standard deviation of the project completion time after mitigation of x risks using ranking index $(\cdot)$ when v simulation iterations are used to compute the project completion time.

$H_{l,v,w}^{(\cdot)_x}$ is the null hypothesis:

$$H_{l,v,w}^{(\cdot)_x} : \mu_{l,v}^{(\cdot)_x} = \mu_{l,w}^{(\cdot)_x}. \tag{33}$$

In other words, for a project l and if x risks are eliminated using ranking index $(\cdot)$, $H_{l,v,w}^{(\cdot)_x}$ predicts that there is no difference in expected project completion time when v rather than w simulation iterations are used to compute the project completion time. If no such difference exists for any value v' (where $v < v' < w$) and if w is sufficiently larger than v, the expected project completion time is said to converge after v simulation iterations.

In order to test $H_{l,v,w}^{(\cdot)_x}$, we adopt a Welch's t-test (Welch, 1947). A Welch's t-test is an adaptation of a Student's t-test and can be used to compare the means of two normally distributed populations that are allowed to have unequal variance. From Welch's t-test, we obtain $p_{l,v,w}^{(\cdot)_x}$, the probability of rejecting $H_{l,v,w}^{(\cdot)_x}$. In addition, define

$$\rho_{v,w,\alpha}^{(\cdot)} = \frac{1}{|L|} \sum_{l \in L} \frac{1}{r} \sum_{x=0}^{r} \delta_{l,v,w,\alpha}^{(\cdot)_x}, \tag{34}$$

where $\delta_{l,v,w,\alpha}^{(\cdot)_x}$ equals 1 if $\left(p_{l,v,w}^{(\cdot)_x} > \alpha \right)$ and 0 otherwise. As such, $\rho_{v,w,\alpha}^{(\cdot)}$ represents the proportion of projects for which the null hypothesis is rejected at an α level of significance.

The number of simulation iterations that is used to compute the ranking values determines the ranking of risks and hence the project completion time at various steps in the mitigation process. Therefore, the rejection of $H_{l,v,w}^{(\cdot)_x}$ does not only depend on the number of simulation iterations itself, but also on the ranking of risks. This, however, does not hold for the random procedure discussed in section 5.2 (i.e., for *RAND*, the ranking of risks does not depend on the number of simulation iterations). Therefore, we use *RAND* to determine the ranking of risks in order to observe only the effect of the number of simulation iterations on the project completion time.
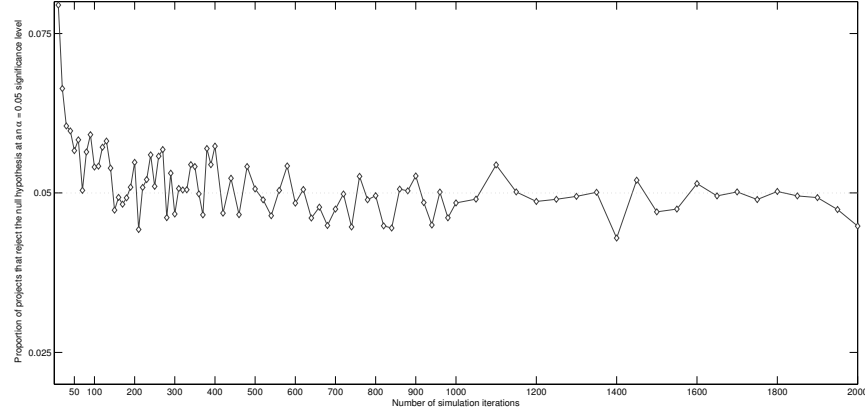
Figure 11: Proportion of projects that rejects the null hypothesis of equal means

Figure 11 presents the proportion of projects that reject $H_{l,v,w}^{(RAND)_x}$ at a 0.05 level of significance, for all $l \in L$, for various values of v and for w equal to 4000. In addition, figure 12 presents the half-width of the confidence interval around the expected project completion time (expressed as a percentage of the mean).

For v larger or equal than 100, we observe that the proportion of projects that rejects the null hypothesis approximates the probability of a type I error (i.e., $\rho_{v,4000,0.05}^{(RAND)}$ approximates 0.05 for v larger or equal than 100). In addition, as of 100 simulation iterations, the half-width becomes sufficiently small such that accurate results may be obtained. As such, we conclude that the project completion time converges starting from 100 simulation iterations if the ranking of risks does not depend on the number of simulation iterations.

5.4.2 Convergence in ranking values

In the previous section, we have shown that 100 simulation iterations suffice to obtain convergence in the completion time of a project if risks are ranked randomly. In this section, we assess the convergence in project completion times if risks are ranked using the ranking indices discussed in the previous sections. If project completion times converge, the ranking of risks (and hence the ranking values themselves) converges as well. Conversely, if project
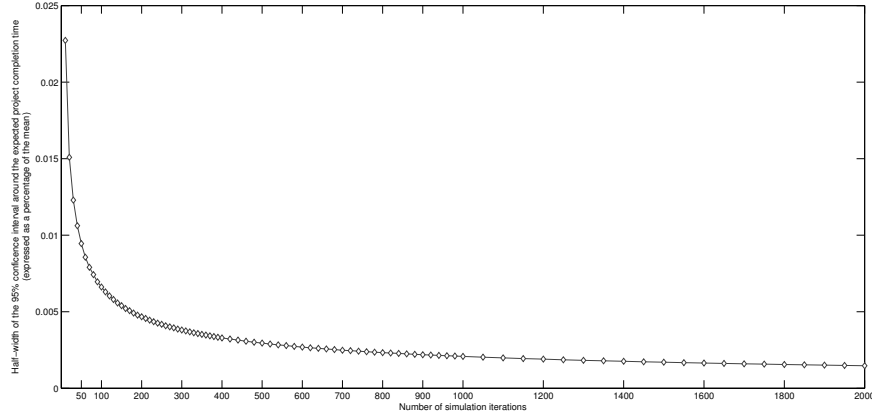
35

Figure 12: Average half-width of the 95 percent confidence interval around the expected project completion time

completion times do not converge, the ranking of risks does not converge either.

Table 7 presents $\rho_{v,1000,0.05}^{(\cdot)}$ for all ranking indices defined in section 4 and for various values of v.

From table 7, we observe that only few ranking indices are able to obtain convergence in less than 1000 simulation iterations. As already discussed in the previous section, the random procedure obtains convergence after 100 simulation iterations. The greedy-optimal procedure takes a bit longer. However, as project completion times converge, the greedy-optimal ranking of risks converges as well. Activity-based ranking indices (in general) as well as correlation-based ranking indices (in particular) have a much harder job in obtaining convergence. Whereas risk-driven ranking indices that use correlation seem to benefit from additional simulation iterations, activity-based ranking show only limited improvement if the number of simulation iterations is increased. This may be explained by the fact that there are more activities than there are risks and that therefore a ranking of activities is more volatile than a ranking of risks. In addition, this effect is further amplified because a ranking of activities has to be mapped onto a ranking of risks (i.e., a two-step procedure is required in order to obtain the set of highest-ranked risks when using an activity-based ranking index).

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Index	$\rho^{(\cdot)}_{v,1000,0.05}$								
RAND	.05	.05	.05	.05	.05	.05	.05	.04	.05
OPT	.09	.06	.04	.03	.03	.03	.03	.03	.03
CA	.04	.04	.04	.04	.04	.04	.04	.04	.04
ACI	.13	.12	.11	.11	.11	.12	.11	.11	.11
SI	.13	.12	.12	.11	.11	.12	.10	.11	.11
CRI	.33	.30	.30	.29	.29	.29	.28	.29	.28
SRCA	.25	.25	.26	.27	.26	.27	.27	.27	.27
SSI	.22	.20	.19	.19	.19	.19	.19	.18	.19
CDCA	.17	.17	.17	.17	.17	.17	.18	.18	.18
CRIR	.47	.38	.34	.33	.32	.31	.31	.31	.31
SRCR	.49	.39	.33	.31	.29	.29	.29	.29	.29
CDCR	.09	.07	.05	.05	.06	.05	.05	.05	.05
v	100	200	300	400	500	600	700	800	900

Table 7: Proportion of projects for which the null hypothesis is rejected for different numbers of simulation iterations

More importantly, however, are the ranking indices that do obtain convergence. Next to CDCR, CA is the only ranking index that is able to obtain convergence in less than 1000 simulation iterations. This does not come as a surprise. CA ranks activities based on whether they are critical in the deterministic schedule ς. As such, the ranking of risks does not depend on the number of simulation iterations and converges at a rate that is equal to the rate that applies when using a random ranking procedure. Even though CA obtains convergence quite rapidly, its performance with respect to mitigating risks is much less to be desired. The performance of CDCR on the other hand nearly matches that of the greedy-optimal procedure. In addition, CDCR is able to obtain convergence in project completion times in less than 1000 simulation iterations, making it the only ranking index that can be used with confidence in practice (i.e., CDCR is able to deliver accurate results with a minimum of computational effort).

Notwithstanding the lack of convergence in most ranking values themselves, the MEI of each of the ranking indices does seem to converge. Table 8 presents the MEI for each of the ranking indices when different numbers of simulation iterations are used to compute the project completion time (note

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Index	$MEI^{(\cdot)}$									
RAND	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
OPT	.57	.57	.57	.56	.56	.56	.56	.56	.56	.56
CA	.45	.45	.45	.45	.45	.45	.45	.45	.45	.45
ACI	.48	.48	.48	.48	.48	.48	.48	.48	.48	.48
SI	.49	.48	.48	.48	.48	.48	.48	.48	.48	.48
CRI	.46	.46	.46	.45	.45	.45	.45	.45	.45	.45
SRCA	.49	.50	.49	.49	.49	.49	.49	.49	.49	.49
SSI	.47	.46	.46	.46	.46	.45	.45	.45	.45	.45
CDCA	.47	.47	.46	.46	.46	.46	.46	.46	.46	.46
CRIR	.41	.46	.47	.48	.48	.48	.48	.48	.49	.49
SRCR	.42	.47	.49	.50	.50	.50	.51	.51	.51	.51
CDCR	.57	.57	.56	.56	.56	.56	.56	.56	.56	.56
v	100	200	300	400	500	600	700	800	900	1000

Table 8: Mitigation efficiency of ranking indices for different numbers of simulation iterations

that due to the use of different random numbers, the results for 1000 simulation iterations differ slightly from those presented in table 6). These results indicate that: (1) an increase of the number of simulation iterations (and hence the computational burden) will result in marginal gains and (2) the simulation model presented in this article is valid and accurate.

6   Additional experiments

In this section, we discuss four additional experiments. In a first experiment, we observe what happens if only a proportion of the impact of a risk can be mitigated (i.e., we assume that it is no longer possible to eliminate the impact of a risk entirely). Next, we discuss the effect of multiplicative risk impacts and risk impacts that are subject to noise. In a fourth and final experiment, activities are no longer clustered in activity groups (i.e., we assume a risk uniformity equal to one).

All experiments assess the performance of the ranking indices over the 600 project networks in the PSPLIB J120 data set. Unless mentioned otherwise, we observe 16 risk profiles that are composed of all combinations of

risk uniformity, risk quantity, risk probability and risk impact as defined in section 5.1. We assume that there is no risk correlation.

6.1   Limited mitigation potential

In section 5, we assume that the impact of a risk can be eliminated entirely when focussing the mitigation efforts on that risk. Often, however, it is only possible to mitigate the impact of a risk up to a certain level. When risks cannot be eliminated, the entries of $\mathbf{d}_j^{(E)}$ are computed as follows:

$$d_{j,p}^{(E)} = d_j + \sum_{e \in E} m_{j,e,p} + \sum_{e \in R \setminus E} \upsilon_e m_{j,e,p}, \tag{35}$$

where υ_e is the proportion of the impact of a risk e that cannot be mitigated.

In this experiment, we use equation 35 (instead of equation 7) to determine $\mathbf{d}_j^{(E)}$ and observe what happens if υ_e equals $1/4$ for all $e \in R$ (i.e., we assume that only 75 percent of the impact of a risk can be mitigated). The results are presented in table 9.

From table 9, it is clear that there is a positive relationship between the mitigation potential of a ranking index and its performance. In addition, one can observe that the gap between the performance of the optimal procedure and the performance of most ranking indices has spread even further. This effect is most significant if risk uniformity is high (i.e., when it is more difficult to distinguish between risks). More importantly, however, is that these conclusions do not hold for *CDCR*, whose performance once more matches that of the greedy-optimal procedure.

6.2   Multiplicative risk impacts

In section 5, the impact of a risk is modeled as a fixed extension of the duration of an activity. In the literature on project risk management, however, many different types of risk impacts have been defined (e.g., breakdowns, start-time delays, fixed impacts, proportional impacts etc.). All of these risk impacts can be brought back to two main categories: (1) impacts that have an additive effect on the duration of an activity and (2) impacts that have a multiplicative effect. When activities are subject to risks that have an additive effect, equation 7 can be used to determine $\mathbf{d}_j^{(E)}$. When subject to risks

Index	Avg	MEI$^{(\cdot)}$															
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
RAND	.001	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
OPT	.538	.35	.40	.37	.42	.36	.41	.38	.42	.65	.69	.67	.71	.67	.71	.68	.72
CA	.395	.19	.27	.22	.30	.22	.29	.24	.31	.43	.55	.47	.58	.50	.61	.52	.62
ACI	.482	.27	.33	.29	.34	.29	.33	.30	.34	.61	.66	.62	.67	.64	.68	.65	.69
SI	.463	.24	.32	.27	.33	.27	.33	.28	.34	.56	.64	.58	.66	.61	.67	.62	.68
CRI	.424	.20	.28	.23	.30	.22	.29	.24	.30	.51	.60	.54	.62	.58	.64	.60	.65
SRCA	.444	.23	.30	.25	.32	.27	.33	.28	.34	.51	.60	.54	.62	.59	.66	.61	.67
SSI	.465	.24	.30	.27	.32	.25	.30	.27	.31	.60	.65	.62	.67	.63	.67	.64	.68
CDCA	.490	.25	.32	.28	.34	.28	.34	.30	.35	.62	.68	.64	.69	.65	.70	.67	.71
CRIR	.433	.21	.29	.24	.31	.26	.32	.28	.34	.49	.58	.53	.61	.58	.64	.60	.66
SRCR	.446	.22	.30	.25	.32	.28	.34	.30	.36	.50	.59	.53	.61	.59	.66	.61	.67
CDCR	.537	.34	.40	.37	.42	.36	.41	.38	.42	.65	.69	.67	.71	.67	.71	.68	.72
Risk uniformity		High								Low							
Risk quantity		High				Low				High				Low			
Risk probability		High		Low		High		Low		High		Low		High		Low	
Risk impact		H	L	H	L	H	L	H	L	H	L	H	L	H	L	H	L

Table 9: Mitigation efficiency of the different ranking indices when the mitigation potential is limited

that have a multiplicative effect, the entries of $\mathbf{d}_j^{(E)}$ are computed as follows:

$$d_{j,p}^{(E)} = d_j + \left[\sum_{e \in E} (d_j m_{j,e,p}) - d_j \right]. \tag{36}$$

In this experiment, we observe what happens when risks have a multiplicative impact on activity durations. Table 10 presents the adopted parameter settings as well as the expected impact on the duration of an activity. Note that: (1) 5.5 time units approximates the average duration of an activity in the project networks of the PSPLIB J120 data set and (2) table 10 also provides the parameter settings of the additive risk impacts that were used in section 5. The results of the experiment are presented in table 11.

It is clear that the conclusions presented in section 5 are still valid here. Therefore, we conclude that the performance of a ranking index does not depend on the impact type of a risk.

Additive impact					
Impact	Type	min	most likely	max	Expected impact
High	Type 1	1.0	2.0	9.0	4.0
High	Type 2	0.0	1.0	2.0	1.0
Low	Type 1	0.5	1.0	4.5	2.0
Low	Type 2	0.0	0.5	1.0	0.5
Multiplicative impact (average duration = 5.5)					
Impact	Type	min	most likely	max	Expected impact
High	Type 1	1.25	1.5	2.5	4.125
High	Type 2	1.0	1.25	1.5	1.375
Low	Type 1	1.125	1.25	1.75	2.0625
Low	Type 2	1.0	1.125	1.25	0.6875

Table 10: Parameter settings for additive and multiplicative risk impacts

Index	Avg	MEI$^{(\cdot)}$															
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
RAND	-.002	-.01	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
OPT	.728	.59	.64	.61	.65	.62	.65	.63	.66	.80	.83	.81	.83	.82	.84	.83	.85
CA	.596	.43	.48	.46	.51	.45	.50	.48	.52	.65	.72	.69	.74	.69	.75	.71	.76
ACI	.632	.48	.52	.49	.52	.49	.52	.50	.53	.73	.76	.74	.76	.75	.78	.76	.78
SI	.655	.51	.54	.52	.55	.53	.55	.53	.55	.75	.78	.76	.78	.77	.79	.78	.80
CRI	.678	.52	.57	.55	.59	.54	.58	.56	.59	.76	.80	.78	.81	.78	.81	.80	.82
SRCA	.700	.54	.59	.57	.60	.58	.61	.59	.62	.78	.81	.80	.82	.81	.83	.82	.84
SSI	.682	.53	.57	.55	.59	.54	.58	.57	.60	.77	.80	.78	.81	.79	.81	.80	.82
CDCA	.687	.52	.57	.54	.58	.55	.59	.57	.61	.77	.81	.79	.82	.80	.83	.81	.84
CRIR	.679	.52	.57	.54	.59	.55	.59	.57	.60	.76	.79	.78	.80	.78	.81	.79	.82
SRCR	.708	.55	.60	.58	.62	.59	.63	.61	.64	.78	.81	.80	.82	.81	.83	.82	.84
CDCR	.725	.59	.63	.61	.65	.61	.65	.63	.66	.79	.82	.80	.83	.81	.84	.82	.84
Risk uniformity		High								Low							
Risk quantity		High				Low				High				Low			
Risk probability		High		Low		High		Low		High		Low		High		Low	
Risk impact		H	L	H	L	H	L	H	L	H	L	H	L	H	L	H	L

Table 11: Mitigation efficiency of the different ranking indices when risk impacts are multiplicative

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6.3   Risk impacts subject to noise

In previous sections, we assume that risk probability and risk impact parameters are known (i.e., we assume to have perfect information). Often, however, these parameters need to be estimated and as we all know: estimates are always wrong. Estimators are subject to bias (i.e., the systematic under- or overestimation of a parameter) and noise (i.e., random errors). When disregarding bias, the estimated duration of an activity j during a simulation iteration p, when subject to a set of risks E is given by:

$$\hat{d}_{j,p}^{(E)} = d_j + \sum_{e \in E} \left(1 + \omega u_p\right) m_{j,e,p}, \tag{37}$$

where ω is the level of noise applied and u_p is a random variate of a random variable U that is uniformly distributed with minimum -1 and maximum 1. In addition, let $\hat{\mathbf{d}}_j^{(E)} = \left\{\hat{d}_{j,1}^{(E)}, \hat{d}_{j,2}^{(E)}, \ldots, \hat{d}_{j,q}^{(E)}\right\}$ denote the vector of estimates that corresponds to the vector of realized durations $\mathbf{d}_j^{(E)}$.

In this experiment, we use the estimated durations (i.e., $\hat{\mathbf{d}}_j^{(E)}$) to determine the ranking values of the different ranking indices. In order to compute the project completion time (i.e., $\mathbf{c}^{(E)}$), we use the realized durations $\mathbf{d}_j^{(E)}$. As such, the decision of which risk to select is based on the estimates whereas the effect of the risk selection itself is determined using the real risk impacts. We assume a deviation of up to 25 percent between the real risk impact and its estimate (i.e., ω equals $1/4$). The results of the experiment are presented in table 12.

When comparing with the results in table 6, it is clear that there is virtually no difference in the performance of the ranking indices if risk impacts are subject to noise. This can be explained by the fact that the expected risk impact (with or without noise) is the same. If bias were to be introduced, we conjecture that the performance of all ranking indices would degrade.

6.4   Unity risk uniformity

In previous sections, we assume that a risk impacts only the activities that are members of one and the same activity group. Therefore, the total expected impact of a risk depends on the size of the impacted activity group (i.e., risks impacting large activity groups have a larger total expected impact than risks that impact only a small activity group). Whereas risk-driven

Index	Avg	MEI$^{(\cdot)}$															
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
RAND	.000	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
OPT	.698	.57	.60	.59	.61	.59	.61	.60	.62	.77	.79	.78	.80	.79	.81	.80	.82
CA	.620	.46	.50	.49	.52	.48	.52	.50	.53	.69	.75	.72	.76	.73	.77	.74	.78
ACI	.642	.49	.52	.51	.53	.51	.53	.52	.53	.74	.76	.75	.77	.77	.78	.77	.79
SI	.641	.49	.52	.51	.53	.51	.53	.52	.53	.74	.76	.75	.77	.76	.78	.77	.79
CRI	.612	.45	.49	.47	.50	.45	.49	.47	.50	.71	.74	.73	.75	.74	.76	.75	.77
SRCA	.657	.49	.53	.51	.54	.52	.55	.53	.56	.75	.78	.77	.79	.78	.81	.79	.81
SSI	.615	.46	.48	.48	.50	.46	.48	.48	.49	.73	.75	.74	.76	.75	.77	.76	.77
CDCA	.645	.47	.51	.49	.52	.50	.53	.51	.54	.75	.78	.76	.79	.78	.80	.79	.81
CRIR	.639	.49	.53	.51	.54	.52	.55	.53	.55	.72	.75	.74	.76	.75	.77	.76	.78
SRCR	.673	.52	.56	.54	.57	.55	.58	.57	.60	.75	.78	.77	.79	.78	.80	.80	.81
CDCR	.696	.57	.60	.58	.61	.58	.61	.60	.62	.77	.79	.78	.80	.79	.81	.80	.82
Risk uniformity		High								Low							
Risk quantity		High				Low				High				Low			
Risk probability		High		Low		High		Low		High		Low		High		Low	
Risk impact		H	L	H	L	H	L	H	L	H	L	H	L	H	L	H	L

Table 12: Mitigation efficiency of the different ranking indices when risk impacts are subject to noise

ranking indices may exploit this information, activity-based ranking indices cannot. Therefore, one might argue that risk-driven ranking indices have an advantage over activity-based ranking indices. In order to offset this advantage, we impose a risk uniformity equal to one. If risk uniformity equals one, all risks impact the same set of activities (i.e., N). As such, no more distinction can be made among risks based on the impacted activity group.

In this experiment, we consider four different risk profiles. For each of these profiles: (1) risk uniformity equals one, (2) risk quantity equals eight, (3) risk probability equals 0.025 and (4) there is no risk correlation. The parameter settings for risk impact are presented in table 13. Note that for all risk profiles the expected impact equals 108 time units (assuming that there are 120 project activities). The results of the experiment are presented in table 14.

From table 14, it is clear that none of the ranking indices is able to distinguish between risks if there is only one risk type. When observing risk profiles 2,3 and 4, however, we see that even though risk-driven ranking

Risk profile 1 (total expected impact = 108 time units)							
Risk type	Number of risks	Risk probability		Risk impact			Expected impact
		min	max	min	most likely	max	
1	8	0.025	0.025	3	4.5	6	108

Risk profile 2 (total expected impact = 108 time units)							
Risk type	Number of risks	Risk probability		Risk impact			Expected impact
		min	max	min	most likely	max	
1	4	0.025	0.025	3	4	5	48
2	4	0.025	0.025	4	5	6	60

Risk profile 3 (total expected impact = 108 time units)							
Risk type	Number of risks	Risk probability		Risk impact			Expected impact
		min	max	min	most likely	max	
1	2	0.025	0.025	2	3	4	18
2	2	0.025	0.025	3	4	5	24
3	2	0.025	0.025	4	5	6	30
4	2	0.025	0.025	5	6	7	36

Risk profile 4 (total expected impact = 108 time units)							
Risk type	Number of risks	Risk probability		Risk impact			Expected impact
		min	max	min	most likely	max	
1	1	0.025	0.025	0	1	2	3
2	1	0.025	0.025	1	2	3	6
3	1	0.025	0.025	2	3	4	9
4	1	0.025	0.025	3	4	5	12
5	1	0.025	0.025	4	5	6	15
6	1	0.025	0.025	5	6	7	18
7	1	0.025	0.025	6	7	8	21
8	1	0.025	0.025	7	8	9	24

Table 13: Risk profile parameter settings

Index	MEI$^{(\cdot)}$			
RAND	-.006	-.006	-.006	-.008
OPT	.019	.073	.167	.338
CA	.000	.001	.001	.002
ACI	.001	.003	.004	.002
SI	.001	.002	.006	.005
CRI	.001	.005	.021	.070
SRCA	.001	.006	.025	.074
SSI	.002	.010	.036	.107
CDCA	.002	.009	.036	.111
CRIR	.001	.046	.146	.326
SRCR	.000	.050	.149	.329
CDCR	.017	.072	.167	.338
Risk profile	1	2	3	4

Table 14: Mitigation efficiency of the different ranking indices when risk uniformity equals one

indices do no longer have the advantage discussed previously, they are still able to outperform the activity-based ranking indices.

7   Conclusions

In this article, we introduced a quantitative, new approach to project risk analysis that allows to address the risk response process in a scientifically-sound manner. We have shown that a risk-driven approach is more effective than an activity-based approach when it comes to analyzing risks. Therefore, project risk management should focus on assessing the uncertainty caused by risks themselves (i.e., the root cause) rather than evaluating the uncertainty at the level of activities.

   In addition, we developed two new ranking indices to assist project managers in determining where to focus their risk mitigation efforts. Ranking indices allow to identify the activities (or risks) that contribute most to the delay of a project (popular ranking indices include the criticality index and the significance index). We developed both an activity-based ranking index (that ranks activities) and a risk-driven ranking index (that ranks risks). We

refer to these ranking indices as *CDCA* and *CDCR* respectively. Both ranking indices outperform existing ranking indices, with *CDCR* nearly matching the performance of a greedy-optimal procedure. In addition, *CDCR* is the only effective ranking index that is able to deliver accurate results within reasonable computation times. It is clear that *CDCR* sets a new standard in the field of ranking indices.

Our conclusions are supported by an extensive computational experiment and were proven to be robust for a broad range of parameter settings. The contributions of this article may be summarized as follows: (1) we assess the performance of a wide variety of ranking indices using a large simulation experiment, (2) we develop two new ranking indices that outperform existing ranking indices and (3) we show that risk analysis should be risk-driven rather than activity-based.

Appendix A

The following is a list of notation used in this article:

- $(\cdot)_j^{(E)}$: ranking value of an activity-based ranking index $(\cdot)$ when activities are subject to a set of risks $E : E \subseteq R$.

- $(\cdot)_e^{(E)}$: ranking value of a risk-driven ranking index $(\cdot)$ when activities are subject to a set of risks $E : E \subseteq R$.

- $A = \{(i,j)|i,j \in N\}$: set of arcs.

- $A^{(N(\cdot))}$: subset of N that contains all activities in N that are ranked highest by activity-based ranking index $(\cdot)$.

- $B^{(E(\cdot))}$: subset of E that contains all risks in E that are ranked highest by ranking index $(\cdot)$.

- c : deterministic project completion time.

- C : random variable that represents the project completion time.

- $\mathbf{c} = \{c_1, c_2, \ldots, c_q\}$: vector of random variates of random variable C.

- $\mathbf{c}^{(E)} = \left\{ c_1^{(E)}, c_2^{(E)}, \ldots, c_q^{(E)} \right\}$: vector of random variates of the project completion time when activities are subject to a set of risks $E : E \subseteq R$.

- $c_p^{(E)}$: project completion time during a simulation iteration p when activities are subject to a set of risks $E : E \subseteq R$.

- d_j : deterministic duration of an activity j.

- D_j : random variable that represents the duration of an activity j.

- $\mathbf{d}_j = \{d_{j,1}, d_{j,2}, \ldots, d_{j,q}\}$: vector of random variates of random variable D_j.

- $\mathbf{d}_j^{(E)} = \left\{ d_{j,1}^{(E)}, d_{j,2}^{(E)}, \ldots, d_{j,q}^{(E)} \right\}$: vector of random variates of the duration of an activity j when subject to a set of risks $E : E \subseteq R$.

- $\hat{\mathbf{d}}_j^{(E)} = \left\{ \hat{d}_{j,1}^{(E)}, \hat{d}_{j,2}^{(E)}, \ldots, \hat{d}_{j,q}^{(E)} \right\}$: estimator of $\mathbf{d}_j^{(E)}$.

- $d_{j,p}^{(E)}$: duration of an activity j during a simulation iteration p when subject to a set of risks $E : E \subseteq R$.

- $\hat{d}_{j,p}^{(E)}$: estimator of $d_{j,p}^{(E)}$.

- δ_j : binary variable that equals 1 if activity j is critical in the deterministic early-start schedule and 0 otherwise.

- $\delta_{j,p}^{(E)}$: binary variable that equals 1 if activity j is critical in $\mathfrak{s}_p^{(E)}$ and 0 otherwise.

- $\Delta^{(E)}$: the expected project delay when activities are subject to a set of risks $E : E \subseteq R$.

- $\delta_{l,v,w,\alpha}^{(\cdot)_x}$: binary variable that equals 1 if $H_{l,v,w}^{(\cdot)_x}$ is rejected at an α level of significance and 0 otherwise.

- $\Delta^{(\cdot)_x}$: the expected project delay after mitigation of x risks using ranking index $(\cdot)$.

- E: subset of R.

- f_j : earliest deterministic finish time of an activity j.

- F_j : random variable that represents the earliest finish time of an activity j.

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- $\mathbf{f}_j = \{f_{j,1}, f_{j,2}, \ldots, f_{j,q}\}$: vector of random variates of random variable F_j.

- $\mathbf{f}_j^{(E)} = \left\{ f_{j,1}^{(E)}, f_{j,2}^{(E)}, \ldots, f_{j,q}^{(E)} \right\}$: vector of random variates of the earliest finish time of an activity j when activities are subject to a set of risks $E : E \subseteq R$.

- $f_{j,p}^{(E)}$: earliest finish time of an activity j during a simulation iteration p when activities are subject to a set of risks $E : E \subseteq R$.

- $G = (N, A)$: graph that consists of a set of nodes $N = \{1, 2, \ldots, n\}$ and a set of arcs $A = \{(i, j) | i, j \in N\}$.

- $H_{l,v,w}^{(\cdot)_x}$: null hypothesis of equal expected project completion time if v and w simulation iterations are used to compute the project completion time of a project l.

- L : set of all project networks in the PSPLIB J120 data set.

- λ_t : weight assigned to a scenario π_t.

- $\mathbf{M} = \{M_{j,e} | j \in N \wedge e \in R\}$: set of risk impacts.

- $M_{j,e}$: random variable that represents the risk impact of a risk e on the duration of an activity j.

- $\mathbf{m}_{j,e} = \{m_{j,e,1}, m_{j,e,2}, \ldots, m_{j,e,q}\}$: vector of random variates of $M_{j,e}$.

- $m_{j,e,p}$: impact of a risk e on an activity j during a simulation iteration p.

- $\mathbf{m}_e = \{m_{e,1}, m_{e,2}, \ldots, m_{e,q}\}$: vector of total risk impacts.

- $m_{e,p}$: total risk impact of a risk e over all activities $j \in N$ during a simulation iteration p.

- $\mathrm{MEI}^{(\cdot)}$: Mitigation Efficiency Index of a ranking index $(\cdot)$.

- $\mu_{l,v}^{(\cdot)_x}$: expected project completion time after mitigation of x risks using ranking index $(\cdot)$ when v simulation iterations are used to compute the project completion time.

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- $N = \{1, 2, \ldots, n\}$: set of nodes.

- ω : maximum deviation between real risk impacts and their estimates.

- $p_{l,v,w}^{(\cdot)_x}$: probability of rejecting $H_{l,v,w}^{(\cdot)_x}$.

- π_t : scenario evaluated at a step t that is characterized by a weight λ_t and a set of risks $E^t : E^t \subseteq R$.

- Π : the set of all scenarios.

- q : number of simulation iterations.

- $R = \{1, 2, \ldots, r\}$: set of all risks.

- $\mathrm{RRD}^{(E(\cdot)_x)}$: the Relative Residual Delay after mitigation of x risks using ranking index $(\cdot)$.

- $\rho_{v,w,\alpha}^{(\cdot)}$: the proportion of projects for which the null hypothesis of equal means is rejected at an α level of significance.

- s_j : earliest deterministic start time of an activity j.

- S_j : random variable that represents the earliest start time of an activity j.

- $\mathbf{s}_j = \{s_{j,1}, s_{j,2}, \ldots, s_{j,q}\}$: vector of random variates of random variable S_j.

- $\mathbf{s}_j^{(E)} = \left\{ s_{j,1}^{(E)}, s_{j,2}^{(E)}, \ldots, s_{j,q}^{(E)} \right\}$: vector of random variates of the earliest start time of an activity j when activities are subject to a set of risks $E : E \subseteq R$.

- $s_{j,p}^{(E)}$: earliest start time of an activity j during a simulation iteration p when activities are subject to a set of risks $E : E \subseteq R$.

- $\mathfrak{s}$: vector of earliest start times when activity durations are deterministic.

- $\mathfrak{s}_p^{(E)}$: vector of earliest starting times during a simulation iteration p when activities are subject to a set of risks $E : E \subseteq R$.

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- $\sigma_{l,v}^{(\cdot)_x}$: standard deviation of the project completion time after mitigation of x risks using ranking index $(\cdot)$ when v simulation iterations are used to compute the project completion time.

- $\mathrm{TF}_{j,p}^{(E)}$: the total float of an activity j during a simulation iteration p when activities are subject to a set of risks $E : E \subseteq R$.

- U : a random variable that is uniformly distributed with minimum -1 and maximum 1.

- u_p :a random variate of a random variable U.

- v_e : proportion of a risk e that cannot be mitigated.

- $\mathbf{y}_j^{(E)} = \left\{ \delta_{j,1}^{(E)}, \delta_{j,2}^{(E)}, \ldots, \delta_{j,q}^{(E)} \right\}$: vector of random variates of the criticality of an activity j when activities are subject to a set of risks $E : E \subseteq R$.

Appendix B

The efficiency of a ranking index may be seen as its ability to correctly identify those risks that have the largest impact on project objectives. As such, for any good ranking index the following holds: $\mathrm{RRD}^{(\cdot)_{x-1}} - \mathrm{RRD}^{(\cdot)_x} \geq \mathrm{RRD}^{(\cdot)_x} - \mathrm{RRD}^{(\cdot)_{x+1}}$ (i.e., $\mathrm{RRD}^{(\cdot)_x}$ has to be convex in the interval $x \in [1, r]$). We illustrate this logic in figure 13. $\mathrm{RRD}^{(\cdot)_x}$ is convex if:

$$\forall x \in [1, r] : \mathrm{RRD}^{(\cdot)_x} \leq \frac{x\mathrm{RRD}^{(\cdot)_r} + \left[(r - x)\, \mathrm{RRD}^{(\cdot)_0} \right]}{r}. \qquad (38)$$

Because $\left(\mathrm{RRD}^{(\cdot)_r} = 0 \right)$ and $\left(\mathrm{RRD}^{(\cdot)_0} = 1 \right)$, the condition translates into:

$$\forall x \in [1, r] : 1 - \frac{x}{r} - \mathrm{RRD}^{(\cdot)_x} \geq 0. \qquad (39)$$

To assess the mitigation efficiency of a ranking index, we want to evaluate the level of convexity of $\mathrm{RRD}^{(\cdot)_x}$. For this purpose, we develop the Mitigation
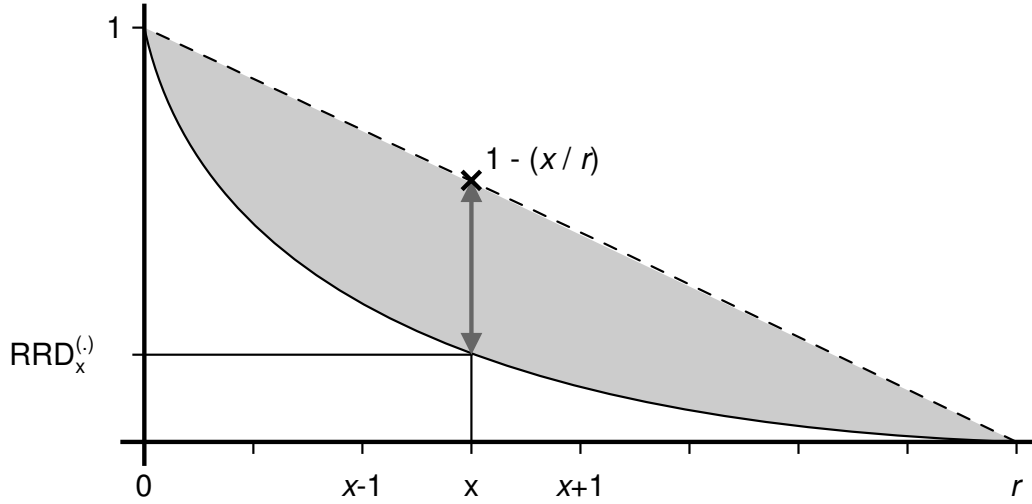
Figure 13: Illustration of mitigation efficiency

Efficiency Index:

$$\widehat{\text{MEI}}^{(\cdot)} \;=\; \sum_{x=1}^{r} 1 - \frac{x}{r} - \text{RRD}^{(\cdot)}{}_x, \tag{40}$$

$$=\; \frac{r-1}{2} - \sum_{x=1}^{r} \text{RRD}^{(\cdot)}{}_x, \tag{41}$$

which corresponds to the surface of the gray area in the graph presented in figure 13. In order to obtain a relative measure, we divide $\widehat{\text{MEI}}^{(\cdot)}$ by $\left(\frac{r-1}{2}\right)$:

$$\text{MEI}^{(\cdot)} = 1 - 2\frac{\displaystyle\sum_{x=1}^{r} \text{RRD}^{(\cdot)}{}_x}{r-1}. \tag{42}$$

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